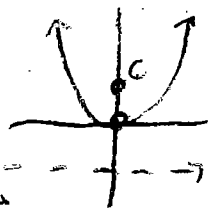


# Parabola

## Standard Equations

$$x^2 = 4py$$

$$p > 0$$



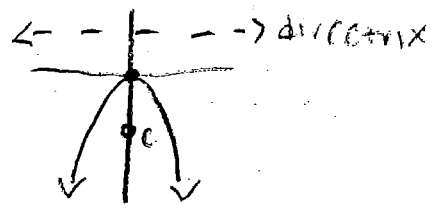
directrix

Vertex: (0,0)

(c) Focus: (0,p)

Directrix:  $y = -p$

$$p < 0$$



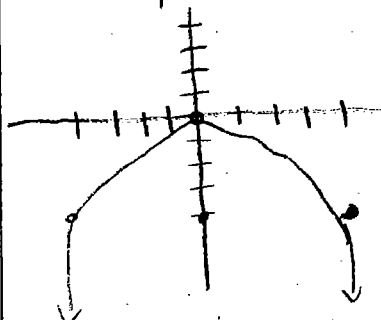
directrix

Example 1: Graph. Find the focus and directrix

(A)

$$x^2 = -16y$$

negative &  
 $x^2$  opens down



focus: (0,-4)

directrix:  $y = 4$

$$4p = -16$$

$$p = -4$$

distance to focus  
& directrix = 4

x	$\pm 4$
y	-1

$$\text{let } y = -1$$

$$x^2 = -16(-1)$$

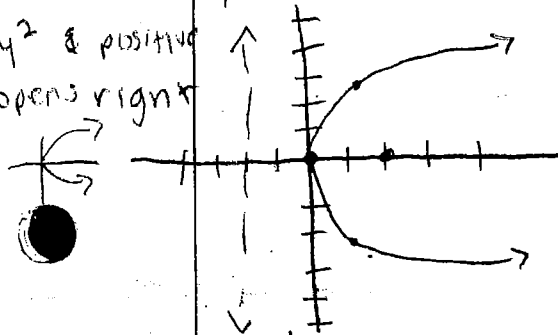
$$x^2 = 16$$

$$x = \pm 4$$

(B)

$$y^2 = 8x$$

$y^2$  & positive  
opens right



$$4p = 8$$

$$p = 2$$

x	1
y	$\pm 2\sqrt{2}$

$$\text{let } x = 1$$

$$y^2 = 8(1)$$

$$y^2 = 8$$

$$y = \pm \sqrt{8}$$

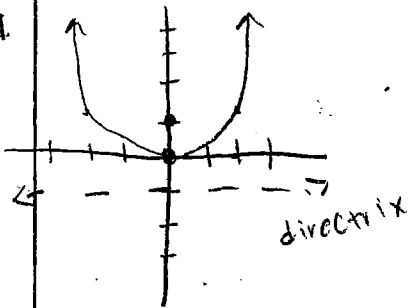
$$y = \pm 2\sqrt{2}$$

focus: (2,0)

directrix:  $x = -2$

positive &  
 $x^2$   
 opens up

(C)  $x^2 = 4y$



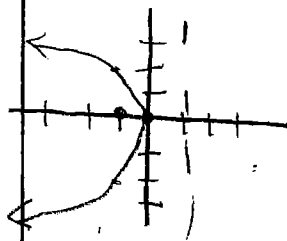
$4p = 4$   
 $p = 1$  distance to focus & directrix = 1

$y = 1$   
 $x^2 = 4(1)$   
 $x^2 = 4$   
 $x = \pm 2$

focus:  $(0, 1)$   
 directrix:  $y = -1$

$y^2$  & neg.

(D)  $y^2 = -4x$



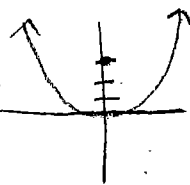
$-4p = -4$   
 $p = 1$

let  $x = -1$   
 $y^2 = -4(-1)$   
 $y^2 = 4$   
 $y = \pm 2$

focus:  $(-1, 0)$   
 directrix:  $x = 1$

Example 2: Find the equation of the parabolas with vertex at  $(0,0)$  with the given info.

(A) focus:  $(0, 3)$



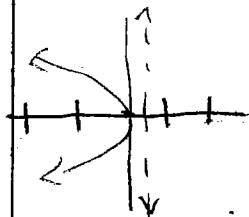
must open up  
 standard equation  $\Rightarrow x^2 = 4py$

$p =$  distance to foci  $\Rightarrow p = 3$

Equation:  $x^2 = 4(3)y$

$x^2 = 12y$

(B) directrix:  $x = 1/2$



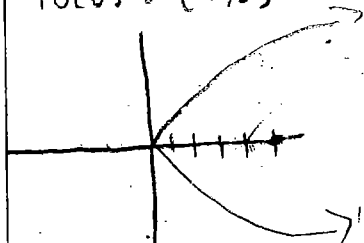
must open left

standard equation  $\Rightarrow y^2 = 4px$   
 $p =$  distance to foci  $\Rightarrow p = -1/2$

$y^2 = 4(-1/2)x$

$y^2 = -2x$

(C) focus: (5,0)



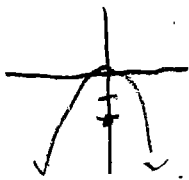
must open right

$$y^2 = 4px$$

$$y^2 = 4(5)x$$

$$y^2 = 20x \quad \checkmark$$

(D) focus: (0,-2)



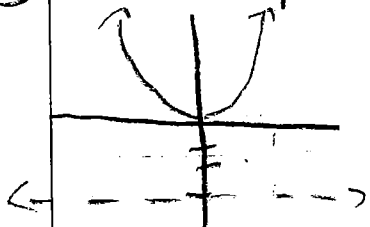
MUST open down

$$x^2 = 4py$$

$$x^2 = 4(-2)y$$

$$x^2 = -8y \quad \checkmark$$

(E) directrix:  $y = -3$



must open up

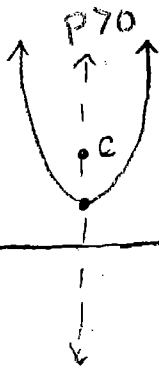
$$x^2 = 4py$$

$$x^2 = 4(3)y$$

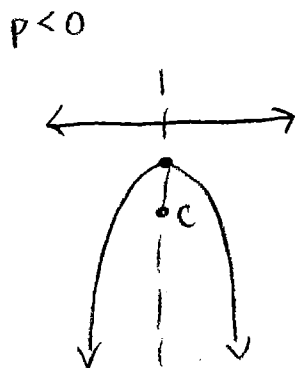
$$x^2 = 12y \quad \checkmark$$

Parabolas cont.

$$(x-h)^2 = 4p(y-k)$$



vertex:  $(h, k)$   
 (c) focus:  $(h, k+p)$   
 axis of symmetry:  $x=h$   
 directrix:  $y=k-p$



Example 4: Find the vertex, focus, directrix, axis of symmetry & graph.

(A)  $(x+3)^2 = -16(y-1)$

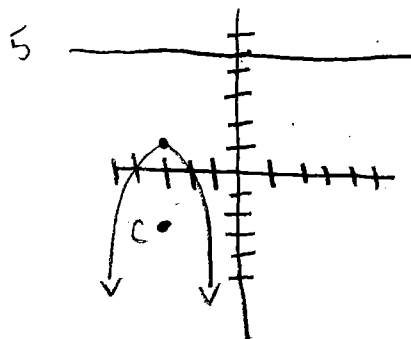
$4p = -16$  opens down  
 $p = -4$

vertex @  $(-3, 1)$

axis of symm.  $x = -3$

focus @  $(-3, 1-4)$   
 $(-3, -3)$

directrix  $y = 1 - (-4)$   
 $y = 5$



(B)  $x^2 + 2x + 4y - 3 = 0$

$$x^2 + 2x = -4y + 3$$

$$x^2 + 2x + \frac{1^2}{2} = -4y + 3 + \frac{1^2}{2}$$

$$(x+1)^2 = -4y + 4$$

$$(x+1)^2 = -4(y-1)$$

$$4p = -4$$

$$p = -1$$

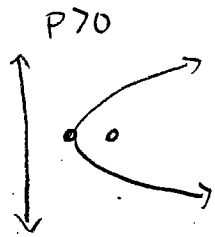
vertex @  $(-1, 1)$

axis:  $x = -1$

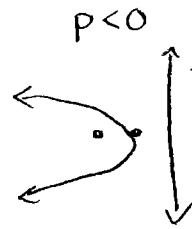
focus:  $(-1, 0)$

directrix:  $y = 2$

$$(y-k)^2 = 4p(x-h)$$

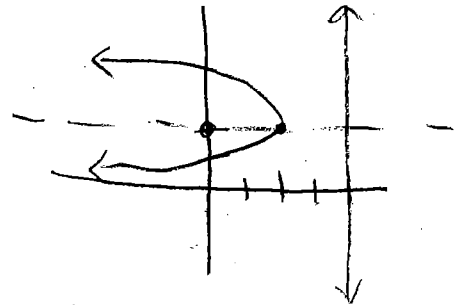


vertex  $(h, k)$   
 focus  $(h+p, k)$   
 axis of symm  $y=k$   
 directrix  $x=h-p$



Example 2: Graph and find all important points.

(A)  $y^2 - 4y + 8x - 12 = 0$   
 $y^2 - 4y + \frac{-2^2}{1} = -8x + 12 + \frac{-2^2}{1}$   
 $\hat{z} = (-2)^2$   
 $y^2 - 4y + 4 = -8x + 16$   
 $(y-2)^2 = -8(x-2)$



open  
left

vertex:  $(2, 2)$

$$4p = -8$$

$$p = -2$$

focus:  $(0, 2)$

directrix  $x=4$

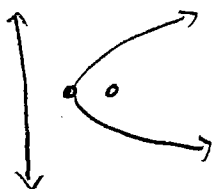
axis of symm  $y=2$

(B)



$$(y-k)^2 = 4p(x-h)$$

$p > 0$



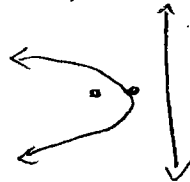
vertex  $(h, k)$

focus  $(h+p, k)$

axis of symm  $y=k$

directrix  $x=h-p$

$p < 0$



Example 2: Graph and find all important points.

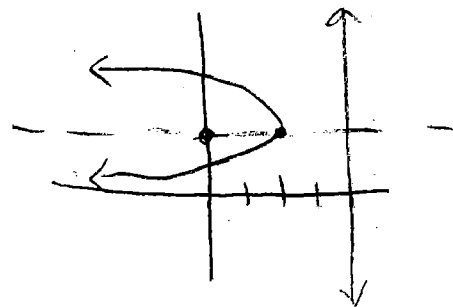
(A)  $y^2 - 4y + 8x - 12 = 0$

$$y^2 - 4y + \frac{-2^2}{2} = -8x + 12 + \frac{-2^2}{2}$$

$$\frac{2}{2} = (-2)^2$$

$$y^2 - 4y + 4 = -8x + 16$$

$$(y-2)^2 = -8(x-2)$$



opens  
left

vertex:  $(2, 2)$

$$4p = -8$$

$$p = -2$$

focus:  $(0, 2)$

directrix  $x=4$

axis of symm  $y=2$

(B)

