

Conics - Ellipse

Ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

center e (h, k)

$\sqrt{\text{denominator of } x} = \text{horizontal distance from center}$

$\sqrt{\text{denominator of } y} = \text{vertical distance from center}$

major axis - long axis

minor axis - short axis

vertices - endpoints of major axis

co-vertices - endpoints of minor axis

Ex. 1 $\frac{(x-2)^2}{36} + \frac{(y+1)^2}{9} = 1$

center e $(2, -1)$

horizontal distance

$$\sqrt{36} = 6$$

$$(2+6, -1) = (8, -1) \left. \vphantom{(2+6, -1)} \right\} \text{vertices}$$

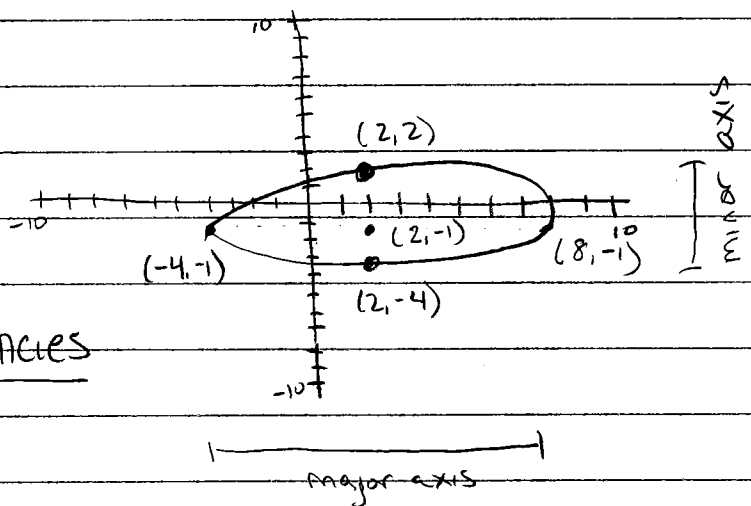
$$(2-6, -1) = (-4, -1) \left. \vphantom{(2-6, -1)} \right\} \text{vertices}$$

vertical distance

$$\sqrt{9} = 3$$

$$(2, -1+3) = (2, 2) \left. \vphantom{(2, -1+3)} \right\} \text{co-vertices}$$

$$(2, -1-3) = (2, -4) \left. \vphantom{(2, -1-3)} \right\} \text{co-vertices}$$



$\boxed{EX 2}$

$$(x+3)^2 + y^2 = 25$$

Center $e (-3, 0)$

horiz distance

$$\sqrt{25} = 5$$

$$(-3+5, 0) = (2, 0)$$

$$(-3-5, 0) = (-8, 0)$$

$\left. \begin{matrix} (-3+5, 0) = (2, 0) \\ (-3-5, 0) = (-8, 0) \end{matrix} \right\}$ co-vertices

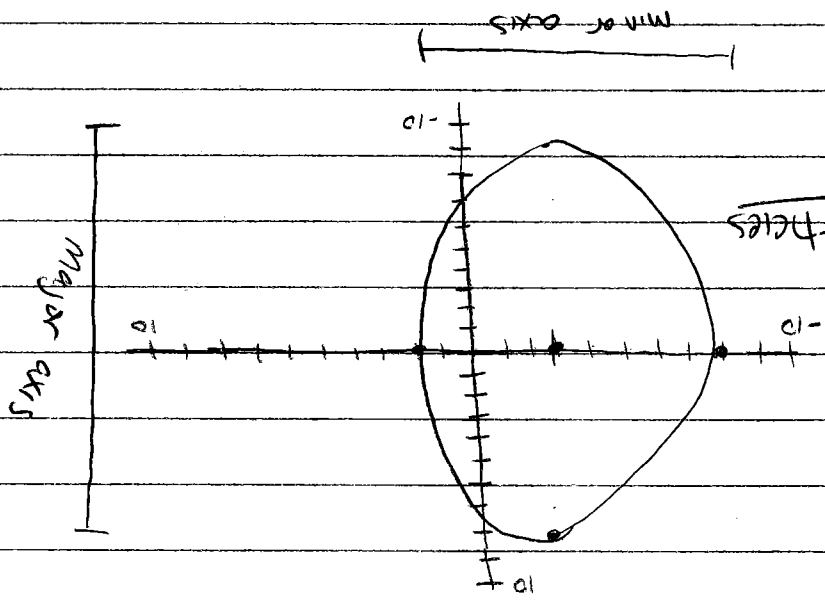
vertical distance

$$\sqrt{64} = 8$$

$$(-3, 0+8) = (-3, 8)$$

$$(-3, 0-8) = (-3, -8)$$

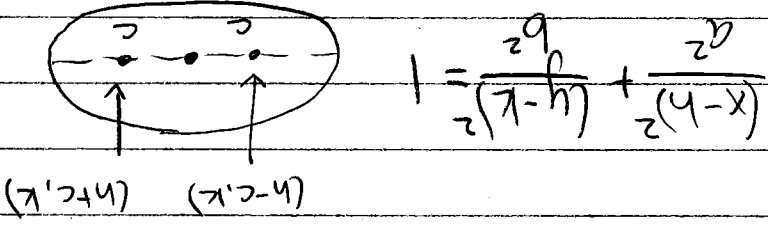
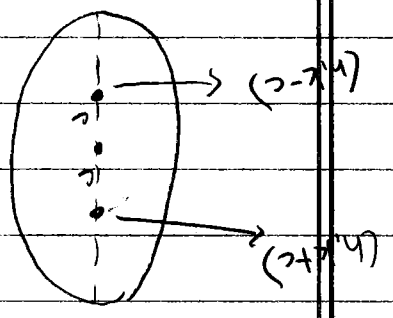
$\left. \begin{matrix} (-3, 0+8) = (-3, 8) \\ (-3, 0-8) = (-3, -8) \end{matrix} \right\}$ vertices



(c) Foci - two points equidistant from the center of an ellipse where the sum of the distance to a fixed point on the ellipse is equal * always on the major axis

$$c^2 = b^2 - a^2$$

$$c^2 = a^2 - b^2$$



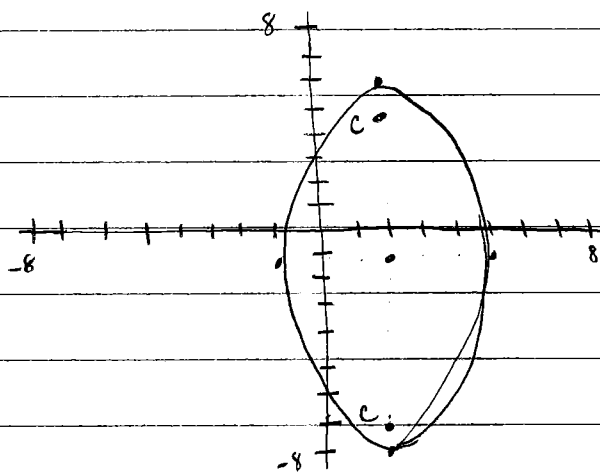
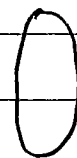
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

* $c^2 =$ larger value - smaller value

Ellipse - cont'd

Ex 3 (a) $\frac{(x-2)^2}{9} + \frac{(y+1)^2}{49} = 1$

* larger value is denominator of y
⇒ ellipse is oriented vertically



center c (2, -1)

horiz. distance $\sqrt{9} = 3$

$$(2+3, -1) = (5, -1)$$

$$(2-3, -1) = (-1, -1)$$

vertical distance $\sqrt{49} = 7$

$$(2, -1+7) = (2, 6)$$

$$(2, -1-7) = (2, -8)$$

major axis - vertical

Vertices: (2, 6) (2, -8)

minor axis - horizontal

co-vertices: (5, -1) (-1, -1)

$$c^2 = 49 - 9$$

$$= 40$$

$$= 2\sqrt{10}$$

foci are on the vertical axis (major axis)

$$(2, -1 + 2\sqrt{10}) = (2, 5.325) \quad \text{foci}$$

$$(2, -1 - 2\sqrt{10}) = (2, -7.325)$$

