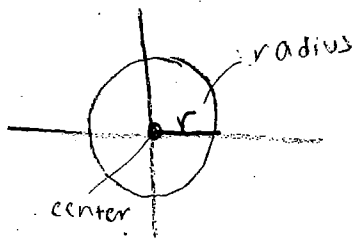


Circles

Standard Equation of a Circle

Center at $(0,0)$ with radius, r



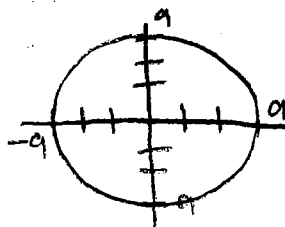
$$x^2 + y^2 = r^2$$

example 1 - Graph & identify the radius

(A) $x^2 + y^2 = 81$

$$r^2 = 81$$

$$\text{radius} = 9$$



(B) $y^2 = 12 - x^2$

*change to standard form

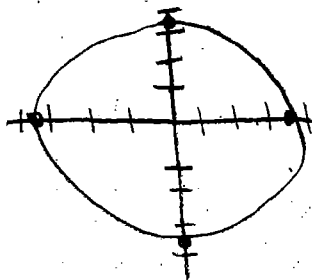
$$\begin{array}{r} +x^2 \quad +x^2 \\ x^2 + y^2 = 12 \end{array}$$

$$r^2 = 12$$

$$r = \sqrt{12}$$

$$r = 2\sqrt{3}$$

$$\boxed{\text{radius} = 2\sqrt{3}}$$



$$\sqrt{12} \approx 3.5$$

Center-Radius form of the Equation of a Circle

center: (h, k) and radius: r

$$(x-h)^2 + (y-k)^2 = r^2$$

example 2: identify the center & radius of each.

(A) $(x+3)^2 + (y-4)^2 = 36$ center: $(-3, 4)$
 $h = -3$ $k = 4$ $r^2 = 36$ radius: 6
 $r = 6$

(B) $(x-2)^2 + (y+5)^2 = 25$ center: $(2, -5)$
radius: 5

(C) $(x+7)^2 + y^2 = 4$ center: $(-7, 0)$
radius: 2

example 3: write the standard form equation of each circle.

(A) center: $(-12, -7)$ $(x+12)^2 + (y+7)^2 = 4$
radius: 2 (h, k)

(B) center: $(4, -5)$ $(x-4)^2 + (y+5)^2 = 25$
radius: 5

(C) center: $(0, 5)$ $x^2 + (y-5)^2 = 81$
radius: 9

(D) center: $(-3, 2)$ $(x+3)^2 + (y-2)^2 = 49$
radius: 7

(E, F)

Circles continued

Completing the square

$$\begin{aligned} \textcircled{1} \quad x^2 + 6x - 3 &= 0 \\ x^2 + 6x &= 3 \\ x^2 + 6x + \underline{3^2} &= 3 + \underline{3^2} \\ \hookrightarrow 6 \div 2 &= 3^2 \\ x^2 + 6x + \underline{9} &= 3 + \underline{9} \\ (x+3)^2 &= 12 \\ x+3 &= \pm \sqrt{12} \\ x+3 &= \pm 2\sqrt{3} \\ \boxed{x = -3 \pm 2\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad x^2 + 8x + 4 &= 0 \\ x^2 + 8x &= -4 \\ x^2 + 8x + \underline{4^2} &= -4 + \underline{4^2} \\ \frac{1}{2} \cdot 8 &= 4^2 \\ x^2 + 8x + \underline{16} &= -4 + \underline{16} \\ x^2 + 8x + 16 &= 12 \\ (x+4)^2 &= 12 \\ x+4 &= \pm \sqrt{12} \\ \boxed{x = -4 \pm 2\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad x^2 + 4x + y^2 - 10y &= 0 \\ (x^2 + 4x + \underline{2^2}) + (y^2 - 10y + \underline{5^2}) &= 0 + \underline{2^2} + \underline{5^2} \\ \frac{1}{2} \cdot 4 &= 2^2 \quad \frac{1}{2} \cdot (-10) = -5^2 \\ (x^2 + 4x + 4) + (y^2 - 10y + 25) &= 29 \\ \text{stop} \quad (x+2)^2 + (y-5)^2 &= 29 \\ \text{Circle center: } &(-2, 5) \\ \text{radius: } &\sqrt{29} \end{aligned}$$

Example 1: Identify the center & radius

(A) $x^2 + 8x + y^2 - 2y - 8 = 0$

$$(x^2 + 8x) + (y^2 - 2y) = 8$$

$$(x^2 + 8x + \underbrace{4^2}_{2=4^2}) + (y^2 - 2y + \underbrace{1^2}_{2=-1^2}) = 8 + 4^2 + \underbrace{-1^2}$$

$$(x^2 + 8x + 16) + (y^2 - 2y + 1) = 8 + 16 + 1$$

$$\star (x+4)^2 + (y-1)^2 = 25$$

center: $(-4, 1)$

$$\text{radius} = \sqrt{25} = 5$$

(B) $x^2 + 10x + y^2 - 4y - 35 = 0$

$$x^2 + 10x + y^2 - 4y = +35$$

$$(x^2 + 10x + \underbrace{5^2}_{2=5^2}) + (y^2 - 4y + \underbrace{2^2}_{2=2^2}) = +35 + \underbrace{5^2} + \underbrace{2^2}$$

$$(x^2 + 10x + 25) + (y^2 - 4y + 4) = +35 + 25 + 4$$

$$\star (x+5)^2 + (y-2)^2 = 64$$

center: $(-5, 2)$

$$\text{radius: } r = 8$$

(C) $6x + 9 + 8y + y^2 + x^2 = 0$

$$x^2 + 6x + y^2 + 8y + 9 = 0$$

center: $(-3, -4)$ $r = 4$

(D) $-4x + x^2 + y^2 + 12y = -4$

$$x^2 - 4x + y^2 + 12y = -4$$

center: $(2, -6)$

$$r = 6$$