AP Stats
Review for the AP Exam - Chapters 6-8

Name: Key
Hour: ______

Chapter 6:

1. What are disjoint (mutually exclusive) events?

Events that cannot occur at the same time

2. How can you prove two events are independent?

$P(A \cap B) = P(A) \cdot P(B)$ OR $P(A|B) = P(A)$

3. A fair coin is flipped 10 times and the number of heads is counted. This procedure of 10 coin flips is repeated 100 times and the results are placed in a frequency table. Which of the frequency tables below is most likely to contain the results from these 100 trials?

(A)

Number of Heads	Frequency
0	19
1	12
2	9
3	6
4	2
5	1
6	3
7	5
8	8
9	14
10	21

(B)

Number of Heads	Frequency
0	9
1	9
2	9
3	9
4	9
5	10
6	9
7	9
8	9
9	9
10	9

(C)

Number of Heads	Frequency
0	0
1	0
2	6
3	9
4	22
5	24
6	18
7	12
8	7
9	2
10	0

(D)

Number of Heads	Frequency
0	7
1	10
2	6
3	11
4	8
5	10
6	9
7	12
8	7
9	11
10	9

(E)

Number of Heads	Frequency
0	0
1	0
2	0
3	2
4	24
5	51
6	22
7	1
8	0
9	0
10	0

4. The probability that a new microwave oven will stop working in less than 2 years is 0.05. The probability that a new microwave oven is damaged during delivery and stops working in less than 2 years is 0.04. The probability that a new microwave oven is damaged during delivery is 0.10. Given that a new microwave oven is damaged during delivery, what is the probability that it stops working in less than 2 years?

(A) 0.05 (B) 0.06 (C) 0.10 (D) 0.40 (E) 0.50

$P(\text{stops working} | \text{damaged}) = \dfrac{P(\text{stops working} \cap \text{damaged})}{P(\text{damaged})} = \dfrac{.04}{.10}$

5. Ninety percent of the people who have a particular disease will have a positive result on a given diagnostic test. Ninety percent of the people who do not have the disease will have a negative result on this test. If 5 percent of a certain population has the disease, what percent of that population would test positive for the disease?

(A) 4.5% (B) 5% (C) 10% (D) 14% (E) 90%

6. Which of the following statements is true for two events, each with probability greater than 0?

(A) If the events are mutually exclusive, they must be independent.
(B) If the events are independent, they must be mutually exclusive.
(C) If the events are not mutually exclusive, they must be independent.
(D) If the events are not independent, they must be mutually exclusive.
(E) If the events are mutually exclusive, they cannot be independent.

Disease

		Yes	No	Total
Test	+	4.5	9.5	14
	-	.5	85.5	86
Total		5	95	100

(1999 #5)

7. Die A has four 9's and two 0's on its faces. Die B has four 3's and two 11's on its faces. When either of these dice is rolled, each face has equal chance of landing on top. Two players are going to play a game. The first player selects a die and rolls it. The second player rolls the remaining die. The winner is the player is the player whose die has the higher number on top.

(a) Suppose you are the first player and you want to win the game. Which die would you select? Justify your answer.

A

B	9	9	9	9	0	0
3	A	A	A	A	B	B
3	A	A	A	A	B	B
3	A	A	A	A	B	B
3	A	A	A	A	B	B
11	B	B	B	B	B	B
11	B	B	B	B	B	B

$P(A \text{ wins}) = \frac{16}{36} = .444$

$P(B \text{ wins}) = \frac{20}{36} = .556$

Die B has a better chance of winning.

(b) Suppose the player using die A receives 45 tokens each time he or she wins the game. How many tokens must the player using die B receive each time he or she wins in order for this to be a fair game? Explain how you found your answer. (A fair game is one in which the player using die A and the player using die B both end up with the same number of tokens in the long run.)

$\frac{16}{36}(45) = \frac{20}{36}(x)$

$36 = x$

Player B needs 36 tokens per win to even the game.

Chapter 7:

8. How do you find the mean and variance of a discrete random variable?

$\mu_x = \sum x_i \cdot p_i$ $\qquad$ $\sigma_x = \sum (x_i - \mu)^2 p_i$

9. What is the Law of Large Numbers:

As the number of observations increases, the observed mean $\bar{x}$ approaches the population mean μ.

10. What are the mean and variance of the sum of two random variables? Of the difference?

$\mu_{x+y} = \mu_x + \mu_y$ $\qquad$ $\mu_{x-y} = \mu_x - \mu_y$

$\sigma^2_{x+y} = \sigma^2_x + \sigma^2_y$ $\qquad$ $\sigma^2_{x-y} = \sigma^2_x + \sigma^2_y$

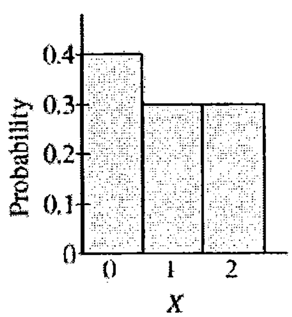
11. A nonprofit organization plans to hold a raffle to raise funds for its operations. A total of 1,000 raffle tickets will be sold for \$1.00 each. After all the tickets are sold, one ticket will be selected at random and its owner will receive \$50.00. The expected value for the net gain for each ticket is -\$0.95. What is the meaning of the expected value in this context?

(A) The ticket owners lose an average of \$0.05 per raffle ticket.
(B) The ticket owners lose an average of \$0.95 per raffle ticket.
(C) Each ticket owner will lose \$0.95 per raffle ticket.
(D) A ticket owner would have to purchase 19 more tickets for the expected value of his or her net gain to increase to \$0.00.
(E) A ticket owner has a 95 percent chance of having a ticket that is not selected.

12. The number of points, X, scored in a game has the probability distributions below.

Sum 0 ⇒ $P(0)\cdot P(0) = .4(.4) = .16$

$P(0)\cdot P(1) + P(1) + P(0) = .3\cdot .4 + .3\cdot .4$
$= .12 + .12$
⇓ Sum 1



Sum

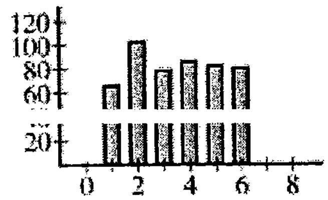
	0	1	2
0	.16	.12	.12
1	.12	.09	.09
2	.12	.09	.09

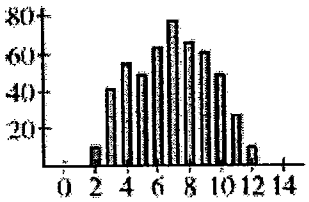
The number of points obtained in one game is independent of the number of points obtained in a second game. When the game is played twice, the sum of the number of points for both times could be 0, 1, 2, 3, or 4. If Y represents the sampling distribution of the sum of the scores when the game is played twice, for which value of Y will the probability be greatest?

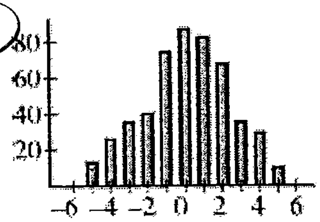
(A) 0 $\quad$ (B) 1 $\quad$ (C) 2 $\quad$ (D) 3 $\quad$ (E) 4

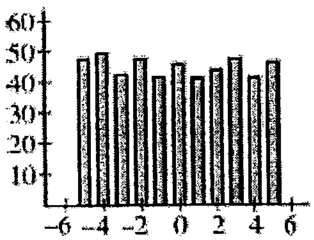
.16 $\quad$ .24 $\quad$ .33 $\quad$ .18 $\quad$ .09

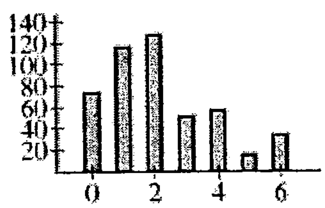
13. For a roll of a fair die, each of the outcomes 1, 2, 3, 4, 5, or 6 is equally likely. A red die and a green die are rolled simultaneously, and the difference of the outcomes (red – green) is computed. This is repeated for a total of 500 rolls of the pair of dice. Which of the following graphs best represents the most reasonable distribution of the differences?

(A)


(B)


(C)


(D)


(E)


Red

Green

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	-1	0	1	2	3	4
3	-2	-1	0	1	2	3
4	-3	-2	-1	0	1	2
5	-4	-3	-2	-1	0	1
6	-5	-4	-3	-2	-1	0

14. Ten percent of all Dynamite Mints candies are orange and 45 percent of all Holiday Mints candies are orange. Two independent random samples, each of size 25, are selected -- one from Dynamite Mints and the other from Holiday Mints candies. The total number of orange candies in the two samples is observed. What are the expected total number of orange candies and the standard deviation for the total number of orange candies, respectively, in the two samples?

(A) 7 and 2.905 (B) 7 and 3.987 (C) 13.75 and 2.233 (D) 13.75 and 2.905 (E) 13.75 and 3.987

$E(X) = \mu_X = 25(.10) + 25(.45) = 13.75$ $\sigma_X = \sqrt{25(.1)(.9) + 25(.45)(.55)} = 2.905$

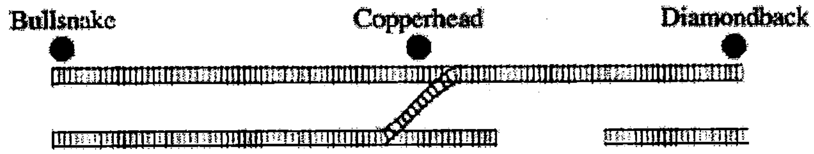
15. The Attila Barbell Company makes bars for weight lifting. The weights of the bars are independent and are normally distributed with a mean of 720 ounces (45 pounds) and a standard deviation of 4 ounces. The bars are shipped 10 in a box to the retailers. The weights of the empty boxes are normally distributed with a mean of 320 ounces and a standard deviation of 8 ounces. The weights of the boxes filled with 10 bars are expected to be normally distributed with a mean of 7,520 ounces and a standard deviation of

(A) $\sqrt{12}$ ounces (B) $\sqrt{80}$ ounces (C) $\sqrt{224}$ ounces (D) 48 ounces (E) $\sqrt{1{,}664}$ ounces

$\sqrt{8^2 + 4^2(10)}$

(2008B #5)

16. Flooding has washed out one of the tracks of the Snake Gulch Railroad. The railroad has two parallel tracks from Bullsnake to Copperhead, but only one usable track from Copperhead to Diamondback, as shown in the figure below. Having only one usable track disrupts the usual schedule. Until it is repaired, the washed-out track will remain unusable. If the train leaving Bullsnake arrives at Copperhead first, it has to wait until the train leaving Diamondback arrives at Copperhead.



Every day at noon a train leaves Bullsnake heading for Diamondback and another leaves Diamondback heading for Bullsnake.

Assume that the length of time, X, it takes the train leaving Bullsnake to get to Copperhead is normally distributed with a mean of 170 minutes and a standard deviation of 20 minutes.

Assume that the length of time, Y, it takes the train leaving Diamondback to get to Copperhead is normally distributed with a mean of 200 minutes and a standard deviation of 10 minutes.

These two travel times are independent.

(a) What is the distribution of $Y-X$?

$\mu_{y-x} = 200-170=30$

$\sigma_{y-x} = \sqrt{10^2+20^2} = 22.361$

The distribution is approximately normal with a mean of 30 minutes and std. dev. of 22.361 min

(b) Over the long run, what proportion of the days will the train from Bullsnake have to wait at Copperhead for the train from Diamondback to arrive?

$P(C>B) = P(Y>X) = P(Y-X>0)$

$z = \frac{0-30}{22.361} = -1.342 \quad P(z>-1.342) = .910$

Approximately 91% of the days, the train will have to wait.

(c) How long should the Snake Gulch Railroad delay the departure of the train from Bullsnake so that the probability that it has to wait is only 0.01?

We will recenter the data by the delay, D. The std. dev would remain 20 min while the mean would shift to 170+D.

$P(y > x+D) = P(y-x-D>0) = P(y-x) - D > 0 = .01$

$\text{invNorm}(.99) = 2.326 \quad z = 2.326 = \frac{0-(30-D)}{22.361}$

$D = 82.012$

The train should be delayed 82.012 minutes.

Chapter 8:

17. What is a binomial distribution?

B - binomial (success / failure)

I - independent events

n - fixed #. of trials

S - probability of success is the same for each trial

18. What is the binomial probability? $P(x=k) = \binom{n}{k} p^k (1-p)^{n-k}$

19. What are the mean and standard deviation of a binomial random variable?

$\mu_x = np$ $\sigma_x = \sqrt{np(1-p)}$

20. What is a geometric distribution?

B - binomial

I - independent

T - Trials continue until a success

S - Probability of success is the same for each trial

21. What are the geometric probabilities?

$P(x=k) = (1-p)^{k-1} \cdot p$ and $P(x>k) = (1-p)^k$

22. What are the mean and standard deviation of a geometric random variable?

$\mu_x = \frac{1}{p}$ $\sigma_x = \frac{\sqrt{1-p}}{p}$

23. The probability of obtaining a head when a certain coin is flipped is about 0.65. Which of the following is closest to the probability that heads would be obtained 15 or fewer times when this coin is flipped 25 times?

(A) 0.14 (B) 0.37 (C) 0.39 (D) 0.60 (E) 0.65

binomcdf (25, .65, 15)

24. In a carnival game, a person can win a prize by guessing which one of 5 identical boxes contains the prize. After each guess, if the prize has been won, a new prize is randomly placed in one of the 5 boxes. If a person makes 4 guesses, what is the probability that the person wins a prize exactly 2 times?

(A) $\frac{2!}{5!}$ (B) $\frac{(0.2)^2}{(0.8)^2}$ (C) $2(0.2)(0.8)$ (D) $(0.2)^2(0.8)^2$ (E) $\binom{4}{2}(0.2)^2(0.8)^2$

25. The probability of winning a certain game is 0.5. If at least 70 percent of the games in a series of n games are won, the player wins a prize. If the possible choices for n are n=10, n=20, and n=100, which value of n should the player choose in order to maximize the probability of winning a prize?

(A) $n = 10$ only
(B) $n = 20$ only
(C) $n = 100$ only
(D) $n = 10$ or $n = 20$ only; the probabilities are the same.
(E) $n = 10$ or $n = 20$ or $n = 100$; the probabilities are the same.

(2004 #3)

26. At an archaeological site that was an ancient swamp, the bones from 20 brontosaur skeletons have been unearthed. The bones do not show any sign of disease or malformation. It is thought that these animals wandered into a deep area of the swamp and became trapped in the swamp bottom. The 20 left femur bones (thigh bones) were located and 4 of these left femurs are to be randomly selected without replacement for DNA testing to determine gender.

(a) Let X be the number out of the 4 selected left femurs that are from males. Based on how these bones were sampled, explain why the distribution of X is not binomial.

Independence requirements are not met since the sampling is done without replacement. This also means the probability of success will not be the same for each trial

(b) Suppose the group of 20 brontosaurs whose remains were found in the swamp had been made up of 10 males and 10 females. What is the probability that all 4 in the sample to be tested are male?

$$\frac{10}{20}\left(\frac{9}{19}\right)\left(\frac{8}{18}\right)\left(\frac{7}{17}\right) = .043$$

(c) The DNA testing revealed that all 4 femurs tested were from males. Based on this result and your answer from part (b), do you think that males and females were equally represented in the group of 20 brontosaurs stuck in the swamp? Explain.

No. If the males and females are equally represented, then the probability of getting all 4 male femurs is only .043. Since this is < .05, it is unlikely they are represented equally.

(d) Is it reasonable to generalize your conclusion in part (c) pertaining to the group of 20 brontosaurs to the population of all brontosaurs? Explain why or why not.

The femurs were all selected from one swamp, so this was not a random sample of femur bones from all brontosaurus.

(2006B #3)

27. Golf balls must meet a set of five standards in order to be used in professional tournaments. One of these standards is distance traveled. When a ball is hit by a mechanical device, Iron Byron, with a 10-degree angle of launch, a backspin of 42 revolutions per second, and a ball velocity of 235 feet per second, the distance the ball travels may not exceed 291.2 yards. Manufacturers want to develop balls that will travel as close to the 291.2 yards as possible without exceeding that distance. A particular manufacturer has determined that the distance traveled for the balls it produces are normally distributed with a standard deviation of 2.8 yards. This manufacturer has a new process that allows it to set the mean distance the ball will travel.

(a) If the manufacturer sets the mean distance traveled to be equal to 288 yards, what is the probability that a ball that is randomly selected for testing will travel too far?

(b) Assume the mean distance traveled is 288 yards and that five balls are independently tested. What is the probability that <u>at least one</u> of the five balls will exceed the maximum distance of 291.2 yards?

(c) If the manufacturer wants to be 99 percent certain that a randomly selected ball will not exceed the maximum distance of 291.2 yards, what is the largest <u>mean</u> that can be used in the manufacturing process?

A $P(x > 291.2) = P\left(z > \frac{291.2-288}{2.8}\right) = P(z > 1.143) = .127$

B $P(x \geq 1) = 1 - P(x=0) = 1 - \left[\binom{5}{0}(.127)^0(1-.125)^5\right]$

C $z^* = 2.326$ $\qquad 2.326 = \frac{291.2-\mu}{2.8}$

invNorm(.99)

$\mu = 284.687$