

1. A t -statistic was used to conduct a test of the null hypothesis $H_0: \mu = 0$ against the alternative $H_a: \mu \neq 0$. The p -value was 0.056. A two-sided confidence interval for μ is to be constructed. Of the following, which is the largest level of confidence for which the confidence interval will NOT contain 0?

- (A) 90% confidence (C) 95% confidence (E) 99% confidence
(B) 93% confidence (D) 98% confidence

2. An automobile manufacturer claims that the average gas mileage of a new model is 35 miles per gallon (mpg). A consumer group is skeptical of this claim and thinks the manufacturer may be overstating the average gas mileage. If μ represents the true average gas mileage for this new model, which of the following gives the null and alternative hypotheses that the consumer group should test?

- (A) $H_0: \mu < 35$ mpg (B) $H_0: \mu \leq 35$ mpg (C) $H_0: \mu = 35$ mpg
 $H_a: \mu \geq 35$ mpg $H_a: \mu > 35$ mpg $H_a: \mu > 35$ mpg
(D) $H_0: \mu = 35$ mpg (E) $H_0: \mu = 35$ mpg
 $H_a: \mu < 35$ mpg $H_a: \mu \neq 35$ mpg

Use the following to answer questions 3-5:

An SRS of 100 postal employees found that the average amount of time these employees had worked for the U.S. Postal Service was $\bar{x} = 7$ years, with a standard deviation of $s = 2$ years. Assume the distribution of the time the population of all postal employees has worked for the Postal Service is approximately normal with mean μ . Do the observed data represent evidence that μ has changed from its value of 7.5 years of 20 years ago? To determine this, we test the hypotheses $H_0: \mu = 7.5$, $H_a: \mu \neq 7.5$ using the one-sample t test.

3. The appropriate degrees of freedom for this test are...

- (A) 9. (B) 10. (C) 19. (D) 99. (E) 100.

4. The p -value for the one-sample t test is

- (A) larger than 0.10.
(B) between 0.05 and 0.10.
(C) between 0.01 and 0.05.
(D) below 0.01.
(E) impossible to determine, since the standard deviation of the study conducted 20 years ago is not given.

$P\text{-value} = .014$

5. Suppose the mean and standard deviation we obtained were based on a sample of 25 postal workers, rather than 100. The p -value would be

- (A) larger.
(B) smaller.
(C) unchanged, since the difference between \bar{x} and the hypothesized value $\mu = 7.5$ is unchanged.
(D) unchanged, since both groups of workers have the same type of job.
(E) unchanged, since the variability measured by the standard deviation stays the same.

6. We wish to see if the dial indicating the oven temperature for a certain model oven is properly calibrated. Four ovens of this model are selected at random. The dial on each oven is set to 300°F . After one hour, the actual temperature of each oven is measured. The temperatures observed are 305° , 310° , 300° , and 305° . Assuming that the actual temperatures for this model when the dial is set to 300° are normally distributed with mean μ , we test whether the dial is properly calibrated by testing the hypotheses $H_0: \mu = 300$, $H_a: \mu \neq 300$. Based on the data, the value of the one-sample t statistic is

- (A) 5. (B) 4.90. (C) 2.82. (D) 2.45 (E) 1.23

7. A radio talk show host with a large audience is interested in the proportion p of adults in his listening area that think the drinking age should be lowered to 18. To find out, he poses the following question to his listeners: "Do you think that the drinking age should be reduced to 18 in light of the fact that 18-year-olds are eligible for military service?" He asks listeners to phone in and vote "yes" if they agree the drinking age should be lowered and "no" if not. You are told that the sample proportion \hat{p} of those who phoned in and answered yes is $\hat{p} = 0.70$ and the standard error of the sample proportion is 0.0459. The number of people who phoned in

- (A) is 21.
 (B) is 50.
 (C) is 100.
 (D) is 200.

$$.0459 = \sqrt{\frac{.7(1-.7)}{n}} \quad n \approx 99.677$$

(E) cannot be determined from the information given.

8. Bags of a certain brand of tortilla chips are claimed to have a net weight of 14 ounces. Net weights actually vary slightly from bag to bag and are normally distributed with mean μ . A representative of a consumer advocate group wishes to see if there is any evidence that the mean net weight is less than advertised and so intends to test the hypotheses $H_0: \mu = 14$, $H_a: \mu < 14$. To do this, he selects 16 bags of this brand at random and determines the net weight of each. He finds the sample mean to be $\bar{x} = 13.88$ ounces and the sample standard deviation to be $s = 0.24$ ounces. Based on the data...

- (A) we would reject H_0 at significance level 0.10 but not at level 0.05.
 (B) we would reject H_0 at significance level 0.05 but not at level 0.025.
 (C) we would reject H_0 at significance level 0.025 but not at level 0.01.
 (D) we would reject H_0 at significance level 0.01 but not at level 0.001.
 (E) we would reject H_0 at significance level 0.001.

$$P\text{-value} = .032$$

9. The analysis of a random sample of 500 households in a suburb of a large city indicates that a 98 percent confidence interval for the mean family income is $(\$41,300, \$58,630)$. Could this information be used to conduct a test of the null hypothesis $H_0: \mu = 40,000$ against the alternative hypothesis $H_a: \mu \neq 40,000$ at the $\alpha = 0.02$ level of significance?

- (A) No, because the value of σ is not known.
 (B) No, because it is not known whether the data are normally distributed.
 (C) No, because the entire data set is needed to do this test.
 (D) Yes, since $\$40,000$ is not contained in the 98 percent confidence interval, the null hypothesis would be rejected in favor of the alternative, and it could be concluded that the mean family income is significantly different from $\$40,000$ at the $\alpha = 0.02$ level.
 (E) Yes, since $\$40,000$ is not contained in the 98 percent confidence interval, the null hypothesis would not be rejected, and it could be concluded that the mean family income is not significantly different from $\$40,000$ at the $\alpha = 0.02$ level.

10. A candy company claims that 10 percent of its candies are blue. A random sample of 200 of these candies is taken, and 16 are found to be blue. Which of the following tests would be most appropriate for establishing whether the candy company needs to change its claim?

- (A) Matched pairs t -test
- (B) One-sample proportion z -test
- (C) Two-sample t -test
- (D) Two-sample proportion z -test
- (E) Chi-square test of association

11. A study was conducted using data collected on the birth weights of a random sample of 10 pairs of identical twins to determine whether the twin born first tends to weigh more than the twin born second. Let μ_F represent the average birth weight of all twins born first, μ_S represent the average birth weight of all twins born second, and μ_D represent the average difference in birth weight (weight of first minus weight of second) for all pairs of twins. Which of the following would be the null and alternative hypotheses for this study?

- (A) $H_0: \mu_F = \mu_S$ and $H_a: \mu_F < \mu_S$
- (B) $H_0: \mu_F = \mu_S$ and $H_a: \mu_F \neq \mu_S$
- (C) $H_0: \mu_D = 0$ and $H_a: \mu_D > 0$
- (D) $H_0: \mu_F - \mu_S = \mu_D$ and $H_a: \mu_F - \mu_S > \mu_D$
- (E) $H_0: \mu_F - \mu_S = \mu_D$ and $H_a: \mu_F - \mu_S \neq \mu_D$

12. A 95% confidence interval for μ is calculated to be (1.7, 3.5). It is now decided to test the hypothesis $H_0: \mu = 0$ versus $H_a: \mu \neq 0$ at the $\alpha = .05$ level, using the same data as used to construct the confidence interval. Which of the following statements is true?

- (A) We cannot test the hypothesis without the original data.
- (B) We cannot test the hypothesis at the $\alpha = .05$ level since the $\alpha = .05$ test is connected to the 97.5% confidence interval
- (C) We can make the connection between hypothesis tests and confidence intervals only if the sample sizes are large.
- (D) We would reject H_0 at the $\alpha = .05$ level.
- (E) We would fail to reject H_0 at the $\alpha = .05$ level.

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13. A large university provides housing for 10 percent of its graduate students to live on campus. The university's housing office thinks that the percentage of graduate students looking for housing on campus may be more than 10 percent. The housing office decides to survey a random sample of graduate students, and 62 of the 481 respondents say that they are looking for housing on campus.

(a) On the basis of the survey data, would you recommend that the housing office consider increasing the amount of housing on campus available to graduate students? Give appropriate evidence to support your recommendation.

① $H_0: p = .10$ $H_a: p > .10$

② One-sample z-test for p where p is the proportion of all graduate students looking for housing on campus at this university.

③ SRS is stated as a random sample of grad students.
 $N \geq 10n = 10(481) = 4810$. We assume there are more than 4810 grad students at this large university, so independence is satisfied.
 $np = 481(.10) = 48.1 \geq 10$ and $n(1-p) = 481(.9) = 432.9 \geq 10$. Since these are both true, the sampling distribution is $\sim N$.

④ $z = \frac{\frac{62}{481} - .1}{\sqrt{\frac{.1(.9)}{481}}} = 2.113$

P-value = $P(z > 2.113) = .0173$

⑤ Because the p-value $.0173 < \alpha = .05$, I reject H_0 . There is sufficient evidence at the .05 level that the proportion of all grad students at this university who are looking for housing on campus is $> 10\%$. The housing office should consider increasing the amount of housing on campus for grad students.

(b) In addition to the 481 graduate students who responded to the survey, there were 19 who did not respond. If these 19 had responded, is it possible that your recommendation would have changed? Explain.

If all 19 had responded "no", then $\hat{p} = \frac{62}{500}$; resulting in $z = 1.789$ and a p-value of .0368. The p-value is still less than .05. So even if all responded no we would still reject H_0 and not change the recommendation.

14. Many suppliers use inspection procedures for quality control. For example, a contract between a manufacturer and a consumer for light bulbs may specify that the mean lifetime of the bulbs must be at least 1000 hours. As part of the quality assurance program, the manufacturer will institute an inspection program for each day's production of 10,000 units. An ordinary testing procedure is difficult since 1000 hours is over 41 days! Since the lifetime of a bulb decreases as the voltage applied increases, a common procedure is to perform an accelerated lifetime test in which the bulbs are lit using 400 volts (compared to the usual 110 volts). At such a voltage, a 1000-hour bulb will last (on average) only 3 hours. This is a well-known procedure, and both sides have agreed that the results from the accelerated test will be a valid indicator of lifetime of the bulb. The manufacturer proposes the following procedure:

Hypotheses: $H_0: \mu_{\text{accelerated}} = 3$
 $H_a: \mu_{\text{accelerated}} < 3$

Sample 100 bulbs at random per day.

Reject all lots whose p -value of the test-statistic is less than 0.05. In other words, the manufacturer will ship a day's production unless the null hypothesis is rejected.

Describe a Type I and a Type II error in this setting. What are the consequences associated with each type of error?

Type I: Reject H_0 that the mean lifetime of all bulbs manufactured that day is 3 hours using 400 volts when the mean really is 3 hours. The consequence of a type I error is the day's production of 10,000 bulbs would be lost unnecessarily, losing money and time.

Type II: Fail to reject H_0 that the mean lifetime of all bulbs manufactured that day is 3 hours using 400 volts when the mean is less than 3 hours. The consequence of a type II error in this case would be a bad shipment of bulbs would be sent out resulting in unsatisfied customers.

