

Directions: Solve each of the following problems, using the available space for scratchwork. Decide which is the best of the choices given and circle the corresponding letter. Do not spend too much time on any one problem.

1. A survey was conducted at a movie theater to determine movie goers' preference for different kinds of popcorn. The results of the survey showed that Brand A was preferred by 65 percent of the people with a margin of error of plus or minus 3 percent. What is meant by the statement "plus or minus 3 percent"?

- (A) Three percent of the population that was surveyed will change their minds.
(B) Three percent of the time the results of such a survey are not accurate.
(C) Three percent of the population was surveyed.
(D) The true proportion of the population who preferred Brand A popcorn could be determined if 3 percent more of the population was surveyed.
(E) It would be unlikely to get the observed sample proportion of 65 percent unless the actual percentage of people in the population of movie-goers who prefer Brand A is between 62 percent and 68 percent.

$$ME = z^* \sqrt{\frac{p(1-p)}{n}}$$

2. A student working on a history project decided to find a 95 percent confidence interval for the difference in mean age at the time of election to office for former American Presidents versus former British Prime Ministers. The student found the ages at the time of election to office for the members of both groups, which included all of the American Presidents and all of the British Prime Ministers, and used a calculator to find the 95 percent confidence interval based on the t -distribution. This procedure is not appropriate in this context because

- (A) the sample sizes for the two groups are not equal
(B) the entire population was measured in both cases, so the actual difference in means can be computed and a confidence interval should not be used
(C) elections to office take place at different intervals in the two countries, so the distribution of ages cannot be the same.
(D) ages at the time of election to office are likely to be skewed rather than bell-shaped, so the assumptions for using this confidence interval formula are not valid.
(E) ages at the time of election to office are likely to have a few large outliers, so the assumptions for using this confidence interval formula are not valid.

3. A simple random sample produces a sample mean, \bar{x} , of 15. A 95 percent confidence interval for the corresponding population mean is 15 ± 3 . Which of the following statements must be true?

- (A) Ninety-five percent of the population ~~measurements~~ fall between 12 and 18.
(B) Ninety-five percent of the sample ~~measurements~~ fall between 12 and 18.
(C) If 100 samples were taken, 95 would ~~fall between~~ 12 and 18.
(D) $P(12 \leq \bar{x} \leq 18) = 0.95$.

(E) If $\mu = 19$, this \bar{x} of 15 would be unlikely to occur.

4. Which of the following is an example of a paired data design?

- (A) A teacher compares the pretest and posttest scores of students.
(B) A teacher compares the scores of students using a computer-based method of instruction with the scores of other students using a traditional method of instruction. *One student using ea. method*
(C) A teacher compares the scores of students in her class on a standardized test with the national average score.
(D) A teacher calculates the average of scores of students on a pair of tests and wishes to see if this average is larger than 80%.
(E) None of these.

10. Many television viewers express doubts about the validity of certain commercials. In an attempt to answer their critics, the Timex Corporation wishes to estimate the proportion of consumers who believe what is shown in Timex television commercials. Let p represent the true proportion of consumers who believe what is shown in Timex television commercials. If Timex has no prior information regarding the true value of p , how many consumers should be included in their sample so that they will be 85% confident that their estimate is within 0.03 of the true value of p ?

(A) 400

(B) 12

(C) 576

(D) 384

(E) 544

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

(2011 #6)

11. Every year, each student in a nationally representative sample is given tests in various subjects. Recently, a random sample of 9,600 twelfth-grade students from the United States were administered a multiple-choice United States history exam. One of the multiple-choice questions is below. (The correct answer is C.)

In 1935 and 1936 the Supreme Court declared that important parts of the New Deal were unconstitutional. President Roosevelt responded by threatening to

(A) impeach several Supreme Court justices.

(B) eliminate the Supreme Court.

(C) appoint additional Supreme Court justices who shared his views.

(D) override the Supreme Court's decisions by gaining three-fourths majorities in both houses of Congress.

Of the 9,600 students, 28 percent answered the multiple-choice question correctly.

(a) Let p be the proportion of all United States twelfth-grade students who would answer the question correctly. Construct and interpret a 99 percent confidence interval for p .

① One sample z CI for p , the proportion of all US 12th graders who would answer correctly.

② Random sample is stated

$n \geq 10(9,600) = 96,000$ since there are more than 96,000 12th graders in the US, independence is satisfied.

$n\hat{p} = 9600(.28) = 2688$ $n(1-\hat{p}) = 9600(.72) = 6912$. Since both are ≥ 10 ,

we can assume Normal approximation

$$\textcircled{3} \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .28 \pm 2.576 \sqrt{\frac{.28(.72)}{9600}} = (.268, .292)$$

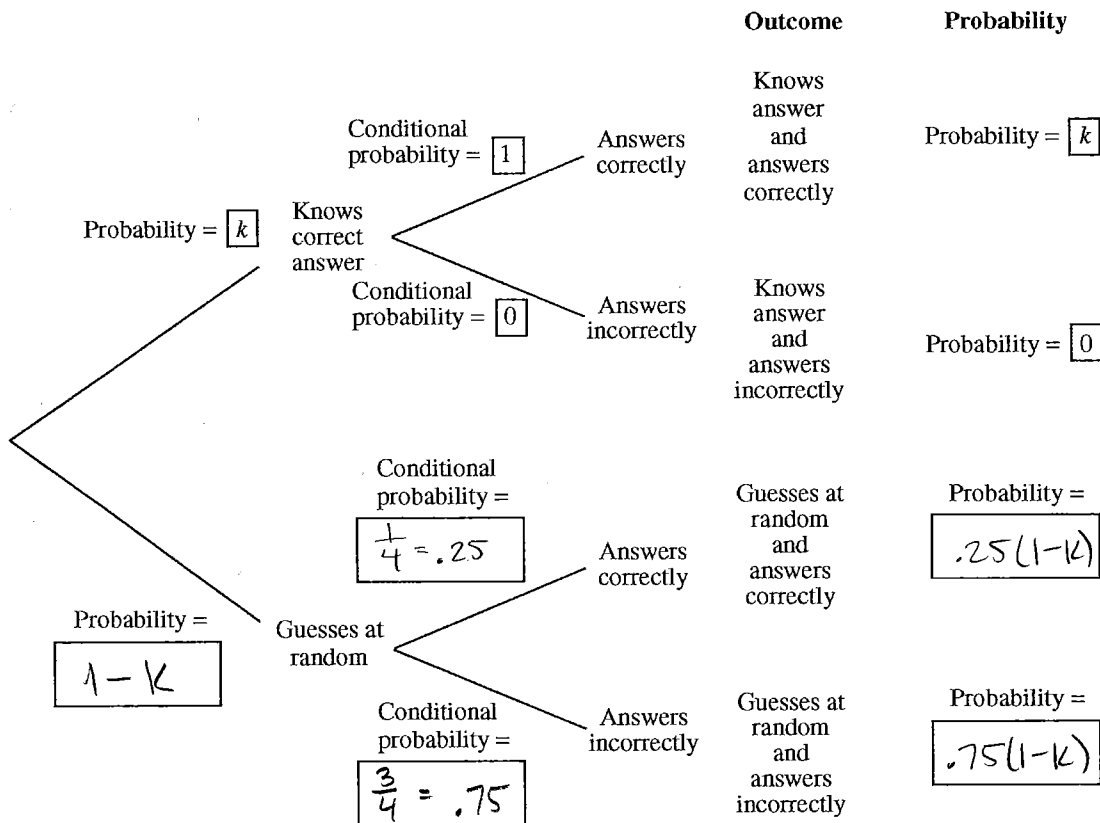
④ Based on this sample, I am 99% confident the true proportion of all U.S. 12th graders who would answer correctly is between .268 and .292.

Assume that students who actually know the correct answer have a 100 percent chance of answering the question correctly, and students who do not know the correct answer to the question guess completely at random from among the four options.

Let k represent the proportion of all United States twelfth-grade students who actually know the correct answer to the question.

(b) A tree diagram of the possible outcomes for a randomly selected twelfth-grade student is provided on the following page. Write the correct probability in each of the five empty boxes. Some of the probabilities may be expressions in terms of k .

TREE DIAGRAM OF OUTCOMES FOR A RANDOMLY SELECTED TWELFTH-GRADE STUDENT



(c) Based on the completed tree diagram, express the probability, in terms of k , that a randomly selected twelfth-grade student would correctly answer the history question.

$$k + .25 - .25k = .75k + .25$$

(d) Using your interval from part (a) and your answer to part (c), calculate and interpret a 99 percent confidence interval for k , the proportion of all United States twelfth-grade students who actually know the answer to the history question. You may assume that the conditions for inference for the confidence interval have been checked and verified.

$$.75k + .25 = .268$$

$$k = .024$$

$$.75k + .25 = .292$$

$$k = .056$$

Based on this sample, I am 99% confident that the true proportion of all US 12th graders who actually know the answer is between .024 and .056.

(1997 #5)

13. A company bakes computer chips in two ovens, oven A and oven B. The chips are randomly assigned to an oven and hundreds of chips are baked each hour. The percentage of defective chips coming from these ovens for each hour of production throughout a day is shown below.

Percentage of Defective Chips

Hour	Oven A	Oven B
1	45	36
2	32	37
3	34	33
4	31	34
5	35	33
6	37	32
7	31	33
8	30	30
9	27	24

The percentage of defective chips produced each hour by oven A has a mean of 33.56 and a standard deviation of 5.20. The percentage of defective chips produced each hour by oven B has a mean of 32.44 and a standard deviation of 3.78. The hourly differences in percentages for oven A minus oven B have a mean of 1.11 and a standard deviation of 4.28.

Does there appear to be a difference between oven A and oven B with respect to the mean percentages of defective chips produced? Give appropriate statistical evidence to support your answer.

① Matched pairs t CI for μ_d , the difference in the mean percentage of all defective chips produced by oven A and oven B (A-B)

② Random assignment of chips is stated, we will assume the hours are also random. $n \geq 10n = 10(9) = 90$, we will also assume there are more than 90 hours of production. The boxplot of the data is approximately normal, so we finally assume the population is N .

$$\textcircled{3} \bar{x} \pm t^* \left(\frac{s}{\sqrt{n}} \right) = 1.11 \pm 2.306 \left(\frac{4.28}{\sqrt{9}} \right) = (-2.183, 4.405) \quad df = 8$$

④ Based on this sample, I am 95% confident that the true difference in the mean percentage of all defective chips produced by these ovens (A-B) is between -2.183 and 4.405%. Since 0 is in the interval, there is NOT evidence of a difference in the percentage of defective chips produced by the ovens.

(2003 #6)

12.