Penny activity

9.3 Sample Means

Example 9.9 Individual stocks vs portfolios

Means of random samples are less variable than individual observations
Means of random samples are more normal than individual observations

Figure 9.15(a) stocks (population) $\mu = -3.5\%$
9.15(b) portfolios (sample) $\bar{x} = -3.5\%$

SRS ~~mean and~~ standard deviation

$$\mu_{\bar{x}} = \mu \qquad \sigma_{x} = \frac{\sigma}{\sqrt{n}}$$

Behavior of $\bar{x}$

- unbiased estimator of μ
- less spread out for large samples
- $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ only when the population is 10x larger than the sample

Example 9.10 The height of young women varies approximately to $N(64.5, 2.5)$.
Meaning $\mu = 64.5$ and $\sigma = 2.5$
Find the mean and std. dev. of an SRS of:
10 women: $\mu_{\bar{x}} = 64.5$ $\sigma_{\bar{x}} = 2.5/\sqrt{10} = .79$ inch
100 women: $\mu_{\bar{x}} = 64.5$ $\sigma_{\bar{x}} = 2.5/\sqrt{100} = .25$ in

Example 9.11

(A) What is the probability a selected woman is taller than 70 inches?

(B) Shorter than 64 inches?

(C) Mean height of 100 women is greater than 66 inches?

(A) $z = \frac{70 - 64.5}{2.5} = 2.2$ $\qquad P(z > 2.2) = .014$

normalcdf(2.2, 100, 0, 1) = .014

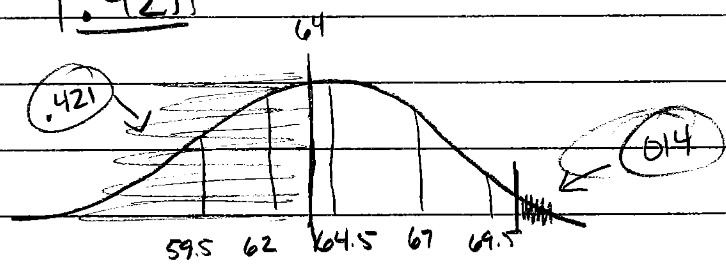
normalcdf(70, 1000, 64.5, 2.5) = .014

$\boxed{.014}$

(B) $z = \frac{64 - 64.5}{2.5} = -.2$ $\qquad P(z < -.2) = .421$

normalcdf(-100, -.2, 0, 1) = .421

$\boxed{.421}$



(C) SRS of 100 $\quad \mu_{\bar{x}} = 64.5 \quad \sigma_{\bar{x}} = \frac{2.5}{\sqrt{100}} = .25$

$z = \frac{66 - 64.5}{.25} = 6$ $\qquad P(z > 6) = 9.9 \times 10^{-10}$

normalcdf(6, 100, 0, 1)

It is highly unlikely we would draw an SRS of 100 women with a mean height greater than 66 inches since the probability is well below 1%.

9.3 cont'd

Exercise 9.31

(a) $\mu_{\bar{x}} = -3.5\%$ $\sigma_{\bar{x}} = 26/\sqrt{5} = 11.628\%$

(b) $P(x>5)$ $z = \frac{5+3.5}{26} = .327$

$P(z>.327) = \boxed{.372}$

(c) $P(\bar{x}>5)$ $z = \frac{5+3.5}{11.628} = .731$

$P(z>.731) = \boxed{.232}$

(D) $P(\bar{x}<0)$ $z = \frac{0+3.5}{11.628} = .301$

$P(z<.301) = \boxed{.618}$

Approximately 62% of all five stock portfolios lost money

Central Limit Theorem - addresses the shape of a sampling distribution $\bar{x}$ when n is sufficiently large ($n \geq 30$). If n is not large enough, the shape closely resembles the original population.

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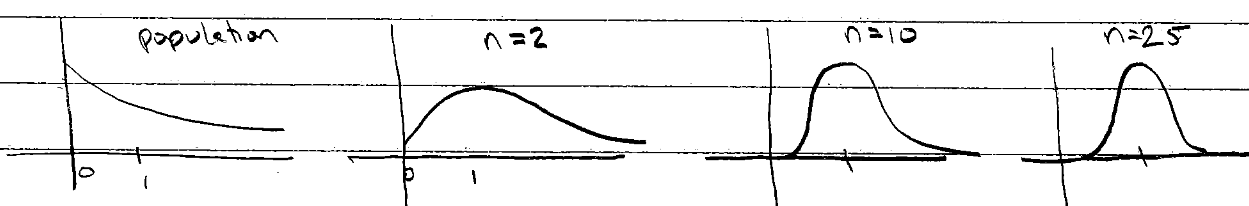

Slide 9

Example 9.13 The time a technician requires to perform preventative maintenance on an air conditioning unit has $\mu = 1$ hour and $\sigma = 1$ hour. The company has a contract to service 70 units. Is it safe to budget an average of 1.1 hours? 1.25 hours?

Sample 70: $\mu = 1$ $\quad \sigma = 1/\sqrt{70} = .120$

(a) $P(\bar{x} > 1.1)$ $\quad z = \dfrac{1.1 - 1}{.120} = .83$ $\quad P(z > .83) = .203$

(b) $P(\bar{x} > 1.25)$ $\quad z = \dfrac{1.25 - 1}{.120} = 2.083$ $\quad P(z > 2.083) = .019$

Conclusion: There is a 20% chance the technician will go over 1.1 hours but only a 2% chance they will go over 1.25 hours. It would be wise to budget 1.25 hr.