8.7 Probability

Pg. 582 3-13 odd, 17-27 odd, 31-36 all

Sample space - all possible outcomes of an experiment

(i.e) tossing a coin: $S = \{H, T\}$

rolling a die: $S = \{1, 2, 3, 4, 5, 6\}$

Probability of Event E

Given a sample space with equally likely outcomes $P(E) = \frac{n(E)}{n(S)}$

$P(E) \Rightarrow$ Probability of event E

$n(E) \Rightarrow$ number of outcomes E

$n(S) \rightarrow$ number of outcomes in the sample space

Example 1 A single die is rolled. Write each event in set notation, and give the probability.

(a) The number showing is 5 or less

$E = \{5, 4, 3, 2, 1\} = 5$

$S = \{6, 5, 4, 3, 2, 1\} = 6$

$P(E) = \boxed{\frac{5}{6}}$

(b) The number showing is odd

$E = \{1, 3, 5\}$

$P(E) = \frac{3}{6} = \boxed{\frac{1}{2}} = .5$ 50% chance of occurring

(c) The number showing is greater than 6.

$E = \emptyset$ (empty set)

$P(E) = \frac{0}{6} = 0$ impossible event

Probability = 0%

(d) less than or equal to 6

$E = \{1, 2, 3, 4, 5, 6\}$

$P(E) = \frac{6}{6} = 1$ event is certain to occur

Probability = 100%

complement (E') - set of all outcomes in the sample space that do not belong to E

(1.e) If $P(E) = \frac{2}{7}$, then $P(E') = 1 - \frac{2}{7} = \frac{5}{7}$

If $P(E) = .64$, then $P(E') = 1 - .64 = .36$

If $P(E) = 21\%$, then $P(E') = 100 - 21 = 79\%$

Odds = $\frac{P(E)}{P(E')}$ = $\frac{\text{favorable outcomes}}{\text{non-favorable outcomes}}$

Example 2 (A) Find the odds of a thrown dice showing 5 or less. (Example 1A)

$\{1, 2, 3, 4, 5, 6\}$

favorable: $\{1, 2, 3, 4, 5\} = 5$

not favorable: $\{6\} = 1$

odds = $\frac{5}{1} = 5:1$ "5 to 1 odds"

(Probability = $\frac{5}{6}$)

8.7 Cont'd

(B) If the probability of the Royals winning the World Series is $\frac{7}{10}$, what are the odds?

$$\frac{\text{favorable}}{\text{not favorable}} = \frac{\text{favorable}}{\text{Total - favorable}} = \frac{7}{10-7} = \boxed{\frac{7}{3}}$$

Probability of the Union of Two Events

$$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

* $P(E \cap F)$ = Probability of E and F

mutually exclusive events - events that cannot occur simultaneously; $P(E \cap F) = \emptyset$

Example 3

(A) A card is draw from a shuffled deck. Find the probability of drawing a Jack or a 10.

$P(\text{Jack}) = \frac{4}{52}$ $P(10) = \frac{4}{52}$ $P(J \cap 10) = \emptyset$

$$P(J \cup 10) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{\frac{2}{13}}$$

(B) Two dice are rolled. Find the probability of each.

(A) The sum is at least 10

$$P(10 \cup 11 \cup 12) = \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{6}{36} = \boxed{\frac{1}{6}}$$

(B) Sum is 2 or both show the same number

sum of 2 is 1+1 ⇒ same #

$$P(2 \cup \text{same \#}) = \frac{1}{36} + \frac{6}{36} - \frac{1}{36} = \frac{6}{36} = \boxed{\frac{1}{6}}$$

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Properties of Probability for any events E and F

1. $0 \le P(E) \le 1$
2. P(certain event) = 1
3. P(an impossible event) = 0
4. $P(E') = 1 - P(E)$
5. $P(E \text{ or } F) = P(E \cup F) = P(E) + P(F) - P(E \cap F)$

(24) (a) $\frac{58,930}{335050} = \boxed{.176}$ (b) $\frac{335050 - 209,117}{335050} = \boxed{.376}$

(c) $\frac{2,744 + 43,571}{335,050} = \boxed{.138}$ (d) $\frac{20,748}{335050 - 20,748} = \boxed{.066}$

Binomial Probability - experiment with only two outcomes in each trial; success or failure

$\binom{n}{r} p^r (1-p)^{n-r}$

n = # of trials

r = # of successes

p = probability of success

1-p = probability of failure

(37-40)

37. $\binom{10}{4}(.69)^4(.31)^6 = \boxed{.042}$

38. $\binom{10}{5}(.11)^5(.89)^5 = \boxed{.002}$

39. P(fewer than 2) = P(0) + P(1)

$= \binom{10}{0}(.44)^0(.56)^{10} + \binom{10}{1}(.44)(.56)^9 = \boxed{.027}$

40. P(no more than 3) = P(0) + P(1) + P(2) + P(3)

$= \binom{10}{0}(.45)^0(.55)^{10} + \binom{10}{1}(.45)^1(.55)^9 + \binom{10}{2}(.45)^2(.55)^8 + \binom{10}{3}(.45)^3(.55)^7$

$= \boxed{.266}$