

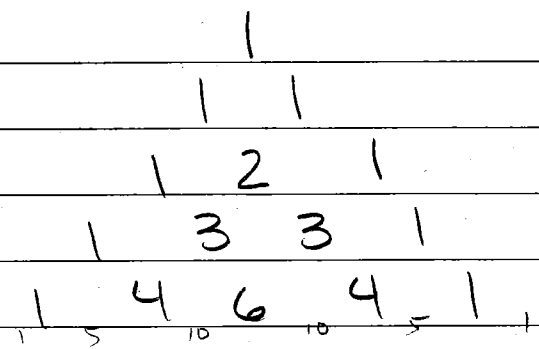
8.5 Binomial Theorem

Pascal's Triangle

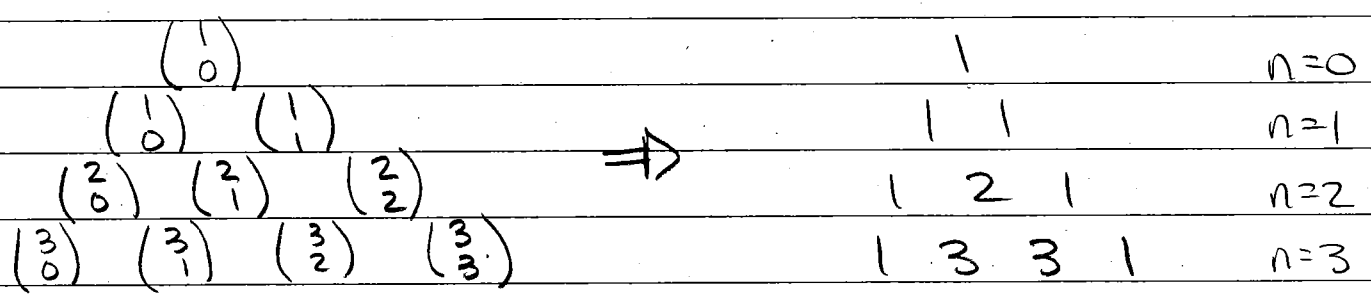
TED video

Secrets of Pascal's

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Binomial Coefficients $n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$



Binomial Expansion

$$(x+y)^0 = 1$$

$$(x+y)^1 = x + y$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Patterns? $(x+y)^n$

Coefficients = Pascal / Binomial Coefficients

Begin e $\binom{n}{0}$ end e $\binom{n}{n}$

First term begins at x^n decreasing to x^0

Second term begins at y^0 decreasing to y^n

Example 1 Determine the binomial expansion

$$\begin{aligned} \text{(A)} \quad (x+y)^6 &= \binom{6}{0}x^6y^0 + \binom{6}{1}x^5y^1 + \binom{6}{2}x^4y^2 + \binom{6}{3}x^3y^3 + \binom{6}{4}x^2y^4 + \\ &\quad \binom{6}{5}x^1y^5 + \binom{6}{6}x^0y^6 \end{aligned}$$

$$= \boxed{x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6}$$

$$\begin{aligned} \text{(B)} \quad (2a+b)^5 &= \binom{5}{0}(2a)^5(b)^0 + \binom{5}{1}(2a)^4(b)^1 + \binom{5}{2}(2a)^3(b)^2 + \binom{5}{3}(2a)^2(b)^3 + \\ &\quad \binom{5}{4}(2a)^1(b)^4 + \binom{5}{5}(2a)^0(b)^5 \end{aligned}$$

$$= (1)(32a^5)(1) + (5)(16a^4)(b) + (10)(8a^3)(b^2) + (10)(4a^2)(b^3) + (5)(2a)(b^4) + (1)(1)(b^5)$$

$$= \boxed{32a^5 + 80a^4b + 80a^3b^2 + 40a^2b^3 + 10ab^4 + b^5}$$

$$\begin{aligned} \text{(C)} \quad (y-3)^4 &= \binom{4}{0}(y)^4(-3)^0 + \binom{4}{1}(y)^3(-3)^1 + \binom{4}{2}(y)^2(-3)^2 + \binom{4}{3}(y)^1(-3)^3 + \\ &\quad \binom{4}{4}(y)^0(-3)^4 \end{aligned}$$

$$= (1)(y^4)(1) + (4)(y^3)(-3) + (6)(y^2)(9) + (4)(y)(-27) + (1)(1)(81)$$

$$= \boxed{y^4 - 12y^3 + 54y^2 - 108y + 81}$$

8.5 cont'd

Example 2 Find the indicated term

(A) 10th term of $(2c-3d)^{14}$ degree
degree \rightarrow $14 = 5 + 9$
 $\binom{14}{9} (2c)^5 (-3d)^9 \leftarrow$ matches $\binom{14}{9}$
one less than
the term

$${}^{14}C_9 2^5 c^5 (-3)^9 d^9 = 2002 \cdot 32 c^5 (-19683) d^9$$

combine

$$\boxed{-1260971712 c^5 d^9}$$

rth term of Binomial Expansion of $(x+ty)^n$

$$\binom{n}{r-1} x^{n-(r-1)} y^{r-1}$$

