

8.4 Counting Theory

Open

Graduation announcements have an option of 4 different color schemes, 3 fonts, and 2 seals. How many different announcements do you have to choose from?

Tree Diagram

$$4 \cdot 3 \cdot 2 = \boxed{24 \text{ choices}}$$

Independent Events - events that do not influence the outcome of the other

with replacement - independent
without replacement - dependent
i.e. Drawing sticks

Example 1 I want to arrange 4 students and assign each a position. Pres, VP, Secretary, Treasurer. Are these events dependent or independent? How many ways can they be arranged
Dependent; $4 \cdot 3 \cdot 2 \cdot 1 = 24$ ways

Suppose I only need a Pres and VP from that same group. How many possibilities are there?

$$4 \cdot 3 = \boxed{12 \text{ possibilities}}$$

n-factorial $n! = n(n-1)(n-2) \dots (3)(2)(1)$
and $0! = 1$

(i.e) $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{120}$
"5 factorial"

Calculator: Scientific - $\boxed{\text{PRB}}$!

Graphing - $\boxed{\text{MATH}}$ $\boxed{\text{PRB}}$!

Permutation - order matters

$$P(n, r) = \frac{n!}{(n-r)!}$$

other notations: P_r^n
 ${}_n P_r$

Repetition
is NOT
Allowed

Combination - order does not matter

$$C(n, r) = \frac{n!}{(n-r)! r!}$$

other notations: C_r^n
 ${}_n C_r$

Permutation

vs.

Combination

From a class of 23 students you must select a VP, Pres, Sec, Treas.

From a class of 23 you must select a committee of 4 students

$${}_{23}P_4 = \frac{23!}{(23-4)!}$$
$$= \frac{23!}{19!}$$

$$= 23 \cdot 22 \cdot 21 \cdot 20 \cdot \cancel{19!}$$

$$= 23 \cdot 22 \cdot 21 \cdot 20$$
$$= \boxed{212520}$$

$${}_{23}C_4 = \frac{23!}{(23-4)! 4!}$$

$$= \frac{23!}{19! 4!}$$

$$= \frac{23 \cdot 22 \cdot 21 \cdot 20 \cdot \cancel{19!}}{\cancel{19!} 4!}$$

$$= 23 \cdot 22 \cdot 21 \cdot 20$$
$$\cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$= \frac{212520}{24} = \boxed{8855}$$

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Calculator: Scientific: $\boxed{\text{PRB}}$ nCr or nPr
Graphing: $\boxed{\text{MATH}}$ $\boxed{\text{PRB}}$ nCr or nPr

Distinguishing between Permutations and Combinations (Chart pg. 559)

- 70
- (a) Codes on a bike lock \Rightarrow permutation
 - (b) Tournament of 12 teams where everyone must play each other once. No home/away advantaged. \Rightarrow Combination
 - (c) telephone number \Rightarrow permutation
 - (d) Sample of 5 apples from a batch of 20 = Com

Examples

(37) 26 letters: 4-letter call letters

$$\text{(a)} \quad \underset{\substack{\uparrow \\ \text{2 options} \\ \text{K or W}}}{2} \cdot \underset{\substack{\uparrow \\ \text{25 options} \\ \text{left}}}{25} \cdot \underset{\substack{\uparrow \\ \text{24 options} \\ \text{left}}}{24} \cdot \underset{\substack{\uparrow \\ \text{23 options} \\ \text{left}}}{23} = \boxed{27,600 \text{ ways}}$$

$$\text{(b)} \quad 2 \cdot 26 \cdot 26 \cdot 26 = \boxed{35152}$$

$$\text{(c)} \quad 2 \cdot 24 \cdot 23 \cdot 1 = \boxed{1104}$$

$$\text{(39)} \quad 5 \cdot 3 = 15 \text{ names}$$

(43) ${}_6P_6 = \boxed{720}$ order matters

or
 $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720}$

(54) ${}_{12}C_3 = \boxed{220}$

(70) $40 \cdot 40 \cdot 40 = 64,000$

* Can't use Permutations because repetition is not allowed in permutations or combinations.