8.3 Geometric Sequence and Series

Open During the month of June you propose an allowance. \$250,000 for the month or 1¢ day 1 and doubling each day.

Day: 1 2 3 4 ... 30

.01 + .02 + .04 + .08 ... 5,368,709.12

Day 30 ⇒ $a_n = a_1 r^{n-1}$ r = common ratio

$= .01(2)^{30-1}$

$= 5368709.12$

Sum ⇒ $S_n = \frac{.01(1-2^{30})}{1-2}$

$= \boxed{10,737,418.23}$

Geometric Sequence - sequence with a common ratio, r $\quad r = \frac{a_{n+1}}{a_n}$

(1e) 2, 4, 8, 16, 32... $\boxed{r=2}$

$\frac{a_2}{a_1} = \frac{4}{2} = 2 \quad \frac{a_3}{a_2} = \frac{8}{4} = 2 \quad \frac{a_4}{a_3} = \frac{16}{8} = 2$

General Term of a Geometric Sequence

$a_n = a_1 r^{n-1}$

(1e) 4, -12, 36, -108 ... $r = \frac{-12}{4} = -3$

$a_n = 4 \cdot -3^{n-1}$ * Do not combine 4 & -3

Example 1 Find a_n and a_{10} for each geometric sequence

(A) $a_1 = 3 \quad r = -5$

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = 3 \cdot (-5)^{n-1}}$

$a_{10} = 3 \cdot (-5)^{10-1}$

$= \boxed{-5859375}$

(B) $\frac{1}{2}, \frac{2}{3}, \frac{8}{9}, \frac{32}{27} \ldots$

$r = \frac{2}{3} / \frac{1}{2} = \frac{2}{3} \cdot \frac{2}{1} = \frac{4}{3}$

$\boxed{a_n = \frac{1}{2} \cdot \left(\frac{4}{3}\right)^{n-1}}$

$a_{10} = \frac{1}{2}\left(\frac{4}{3}\right)^{10-1} = \frac{1}{2}\left(\frac{262144}{19683}\right)$

$\boxed{6.659}$

Example 2 Find a_1 and r for the geometric sequence

$a_2 = -6 \quad a_7 = -192$

Using $a_n = a_1 r^{n-1}$, set up equations for a_2 and a_7

Solve as a system

$a_2 = a_1 r^{2-1}$

$\frac{-6}{r} = \frac{a_1 r}{r}$

$\boxed{\frac{-6}{r}} = a_1$

sub a_1

$a_7 = \boxed{a_1} \cdot r^{7-1}$

$-192 = \frac{-6}{r} \cdot r^6$

$\frac{-192}{-6} = \frac{-6r^5}{-6}$

$\sqrt[5]{32} = \sqrt[5]{r^5}$

$\boxed{2 = r}$

sub r to find a_1

$a_2 = a_1 \cdot 2^{2-1}$

$\frac{-6}{2} = \frac{a_1 \cdot 2}{2}$

$\boxed{a_1 = -3}$

$\boxed{8.3}$ cont'd

Partial Sum of a Geometric Sequence

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Infinite Sum of Geo Sequence

* Only if $|r| < 1$ $\Rightarrow$ Sequence converges
- approaches a single value

$$S_\infty = \frac{a_1}{1-r}$$

Example 3 (A) Find the sum of the first 10 terms
(B) Find the infinite sum, if it exists

(A) $4, 16, 64, 256 \ldots$ $r = 4$

$$S_{10} = \frac{4(1-4^{10})}{1-4} = \frac{4(-1048575)}{-3} = \boxed{1{,}398{,}100}$$

S_∞ does not exist $r = 4 > 1$

(B) $12, -4, \frac{4}{3}, -\frac{4}{9} \ldots$ $r = -\frac{1}{3}$

$$S_{10} = \frac{12(1-(-\frac{1}{3})^{10})}{1-(-\frac{1}{3})} = \frac{12(1-\frac{1}{59049})}{\frac{4}{3}} \approx \boxed{9}$$

$S_\infty \Rightarrow r = -\frac{1}{3}$ $|-\frac{1}{3}| < 1$ infinite sum exists

$$S_\infty = \frac{12}{1-(-\frac{1}{3})} = \frac{12}{\frac{4}{3}} = \frac{12 \cdot 3}{4} = \boxed{9}$$

Example 4 Find each sum (* r is the value taken to a power
$6^n \Rightarrow r = 6$
$\frac{1}{2}\left(\frac{2}{3}\right)^n \Rightarrow r = \frac{2}{3}$)

(A) $\sum_{i=1}^{17} (-2)^i$ $r = -2$
$a_1 = -2$

$S_{17} = \frac{-2(1-(-2)^{17})}{1-(-2)}$

$= \frac{-2(131073)}{3}$

$= \boxed{-87382}$

(B) $\sum_{i=1}^{\infty} 12\left(\frac{3}{2}\right)^i$ $r = \frac{3}{2} > 1$ $\boxed{\text{no infinite sum}}$

(C) $\sum_{i=1}^{\infty} \frac{4}{3}\left(\frac{2}{3}\right)^i$ $r = \frac{2}{3}$ $a_1 = \frac{4}{3} \cdot \frac{2}{3} = \frac{8}{9}$

$S_\infty = \frac{\frac{8}{9}}{1-\frac{2}{3}} = \frac{\frac{8}{9}}{\frac{1}{3}} = \frac{8}{9} \cdot \frac{3}{1} = \boxed{\frac{8}{3}}$

Future Value of an Annuity $\Rightarrow$ formula pg. 550 (57-60)

(67) loses 20% $\Rightarrow$ 80% left $r = .80$
$a_1 = 100{,}000$
$n = 6$ years

$a_n = a_1 \cdot r^{n-1}$
$a_6 = 100{,}000 \cdot .80^{6-1}$
$= 100{,}000 \cdot .80^5$
$= \boxed{32{,}768}$