Pg. 542 3-27 odd, 31, 37-61 EOO
Skip 15, 23

8.2 Arithmetic Sequences & Series

Arithmetic Sequence - Sequence in which each term increases or decreases by a common difference (d)

$-4, -1, 2, 5 \ldots$	$d = 3$
$7, 3, -1, -5 \ldots$	$d = -4$
$2, 4, 8, 16 \ldots$	not arithmetic

Example 1 Find the first five terms

(A) $a_1 = 30 \quad d = -5$

30, 25, 20, 15, 10

(B) $a_3 = -7 \quad d = 3$

$\underline{-13} \quad \underline{-10} \quad \underline{-7} \quad \underline{-4} \quad \underline{-1}$

a_3

General Term of an Arithmetic Sequence

$a_n = a_1 + d(n-1)$

* d = slope

Example 2 Find a_{10} and a_n for each

(A) $a_1 = 6 \quad d = 4$

$a_n = a_1 + d(n-1)$

$a_n = 6 + 4(n-1)$

$a_n = 6 + 4n - 4$

$\boxed{a_n = 2 + 4n}$

$a_{10} = 2 + 4(10)$

$= 2 + 40$

$\boxed{a_{10} = 42}$

(12) $a_8 = 16 \quad a_{14} = 46$

① need d = slope = $\frac{y_1 - y_2}{x_1 - x_2}$

$a_8 = 16 \Rightarrow (8, 16)$

$a_{14} = 46 \Rightarrow (14, 46)$

$d = \frac{16 - 46}{8 - 14} = \frac{-30}{-6} = 5$

② need a_1; use either given point

$a_n = a_1 + d(n-1)$

$a_8 = a_1 + 5(8-1)$

$16 = a_1 + 35$

$-19 = a_1$

$a_n = a_1 + d(n-1)$

$a_n = -19 + 5(n-1)$

$a_n = -19 + 5n - 5$

$\boxed{a_n = -24 + 5n}$

$a_{10} = -24 + 5(10)$

$= -24 + 50$

$\boxed{a_{10} = 26}$

Sum of the First n Terms of an Arithmetic Sequence

$S_n = \frac{n}{2}(a_1 + a_n)$

n = # of terms

a_1 = 1st term

a_n = last term

(02) $S_n = \frac{n}{2}(2a_1 + d(n-1))$

n = # of terms

a_1 = 1st term

d = common difference

Example 3 Find the sum of the first 12 terms ($n = 12$)

(A) $a_1 = 10 \quad d = -3$

$S_{12} = \frac{12}{2}(2 \cdot 10 + -3(12-1))$

$= 6(20 - 33)$

$= 6(-13)$

$= -78$

(B) $8, 6, 4 \ldots \quad d = -2$

$S_{12} = \frac{12}{2}(2(8) + -2(12-1))$

$= 6(16 - 22)$

$= 6(-6)$

$= \boxed{-36}$

18.2 cont'd

© $a_1 = -6 \quad a_{12} = 30$ Given first and last terms

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{12} = \frac{12}{2}(-6 + 30)$

$= 6(24)$

$= 144$

Example 4 Find a_1 and d for the sequence

(A) $S_{31} = 5580 \quad a_{31} = 360$

Sum of 31 terms — last term

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{31} = \frac{31}{2}(a_1 + a_{31})$

$5580 = \frac{31}{2}(a_1 + 360)$

$\frac{31}{2} \qquad \frac{31}{2}$

$360 = a_1 + 360$

$\boxed{0 = a_1}$

$d = \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$

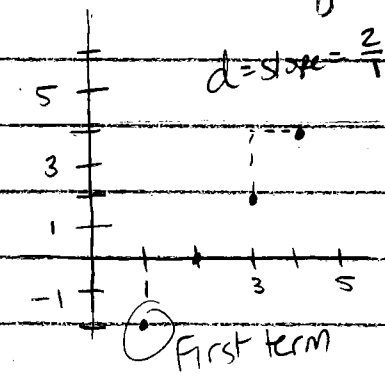
$a_1 = 0 \Rightarrow (1, 0)$

$a_{31} = 360 \Rightarrow (31, 360)$

$d = \frac{360 - 0}{31 - 1} = \frac{360}{30}$

$\boxed{d = 12}$

Example 5 Find a formula for the nth term of the sequence



$a_n = a_1 + d(n-1)$

$a_n = -2 + 2(n-1)$

$a_n = -2 + 2n - 2$

$\boxed{a_n = 2n - 4}$

Example 5 Use a formula to find the sum of the arithmetic series

$89 + 84 + 79 + 74 + \ldots + 9 + 4$

$d = -5$ # of terms?

$a_n = a_1 + d(n-1)$ solve for n using the last term $a_n = 4$

$4 = 89 + -5(n-1)$

$4 = 89 - 5n + 5$

$4 = 94 - 5n$

$5n = 90$

$n = 18$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{18} = \frac{18}{2}(89 + 4)$

$= 9(93)$

$\boxed{S_{18} = 837}$

Example 6 Evaluate the sum

$$\sum_{j=1}^{15} (5j - 9)$$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{15} = \frac{15}{2}(-4 + 66)$

$= 7.5(62)$

$= \boxed{465}$

$n = 15$

$a_1 = 5(1) - 9$

$= -4$

$a_{15} = 5(15) - 9$

$= 66$