Acvity 8A:

Answers:

1. A
2. A
3. E
4. D
5. A
6. E
7. B
8. A
9. E
10. D

The Binomial Distribuon

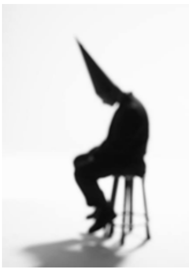
Secon 8.1

Many random variables seen in pracce amount to counng the number of successes in *n* independent observaons, such as:

- The number of doubles in four rolls of a pair of dice.
- The number of paents with type A blood in a random sample of 10 paents.
- The number of defecve items in a sample of 20 items.

These situaons are called “binomial” because each trial has two possible outcomes:

“success” or “failure”



Binomial Seng? - Check for BINS

- B: binomial (Each observaon has only 2 outcomes
  - a success or a failure.)
- I: observaons are all independent (That is, knowing the result of one observaon tells you nothing about the other observaons.)
- N: there is a fixed number of observaons, *n*
- S: the probability of a success, *p*, is the same for each observaon

The binomial distribuons are an important class of discrete probability distribuons .

If you are presented with a random phenomenon, it is important to be able to recognize it as a binomial seng or a geometric seng (the next secon.)

If data is produced in a binomial seng, then the random variable X = the number of successes is called a binomial random variable.

The probability distribuon of X is called a binomial distribuon.

As an abbreviaon, we say that X is $B(n, p)$.

$\sim N(\mu, \sigma)$

Binomial or Not?

1. The pool of potenal jurors for a murder case contains 100 people chosen at random from the adult residents of a large city. Each person in the pool is asked whether he or she opposes the death penalty; X is the number who say "Yes."

yes, binomial.

Binomial or Not?

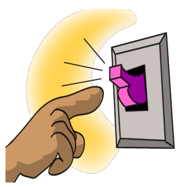
2. At peak periods, 15% of aempted log-ins to an email service fail. Log-in aempts are independent and each has the same probability of failing. Darci logs in repeatedly unl she succeeds. *X* is the number of log-in aempts it took to get through.

no, not binomial - not a fixed number of observaons, *n*.

The binomial distribuons are important in stascs when we wish to make inferences about the proporon *p* of "successes" in a populaon.

A typical example:

An engineer chooses an SRS of 10 switches from a shipment of 10,000 switches. Suppose that (unknown to the engineer) 10% of all the switches are bad. The engineer counts *X* = the number of bad switches in the sample.



The example shows how we can use the binomial distribuons in the stascal seng of selecng an SRS.

When the populaon size is much larger than the sample, a count of successes in an SRS of size *n* has approximately the binomial distribuon *B*(*n*, *p*).

What are my chances then?

The binomial probability that you get exactly $X = k$ successes is given by:

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$$

where the binomial coefficient is:

$$\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$$

r=k

in calc.: n Math PRB 3:nCr K

on Pop Quiz

P(you got 3 right out of 10)

$\binom{10}{3}$ = 10 MATH PRB 3: $_n$Cr 3 ENTER

=120

Each child born to a parcular set of parents has probability 0.25 of having blood type O. If these parents have 5 children, what is the probability that exactly 2 of them have type O blood?

$$\binom{5}{2}(.25)^2(1-.25)^{5-2} \approx .264$$

binompdf(5, .25, 2)

2nd VARS *?:binompdf(n, p, X)*

0
A

n = number of observaons

p = probability of the ~~outcome = P(k)~~ success

X = ~~random variable =~~ k = # of successes

If 25% of all adults are college graduates, find the probability that 3 out of an SRS of 4 adults are all college graduates.

> You must **show** the set-up:

$$\binom{4}{3}(.25)^3(1-.25)^{4-3}$$

> But then use your calculator:

binompdf(4, .25, 3)

$\approx .047$

2nd VARS ?:binomcdf(n, p, X)

B
A

n = number of observaons

p = probability of the outcome = P(k)

X = ~~random variable~~ = k = # of successes

**** This will only give probabilies LESS THAN OR EQUAL TO a specified value!!!***

What if we want the probability that 2 or fewer of our sample of 4 are college graduates?

$\binom{4}{2}(.25)^2(1-.25)^{4-2} + \dots$

$+\binom{4}{0}(.25)^0(1-.25)^{4-0}$

binomcdf$(4,.25,2) \approx .949$

Is this the same as the probability that less than 2 are college graduates? no $P(X<2)=P(X\leq 1)$

What about the probability of 3 or more are college graduates? $1-P(X\leq 2)$

The median annual household income in the US is about $39,000.

Among 5 randomly selected households find the probability that 4 or more have incomes below $39,000 per year.

$\binom{5}{4}(.5)^4(1-.5)^{5-4} +$

$\binom{5}{5}(.5)^5(1-.5)^{5-5}$

$1-$binomcdf$(5,.5,3)$

$=.1875$

Tonight's HW: p. 516 #1, 3, 5-6, 8, 10-12, 14, 16-18

The Mean of a Binomial Random Variable:

$$\mu_x = np$$

The Standard Deviaon of a Binomial Random Variable:

$$\sigma_x = \sqrt{np(1-p)}$$

*Be careful to ONLY use these in binomial situaons.

The median annual household income in the US is about \$39,000. Now consider a random sample of 16 households.

What is the expected number of households with income below \$39,000?

$$E(X) = \mu_X = .5(16) = 8$$

What is the standard deviaon of the number of households with incomes under \$39,000?

$$\sigma_X = \sqrt{16(.5)(1-.5)}$$
$$= 2$$

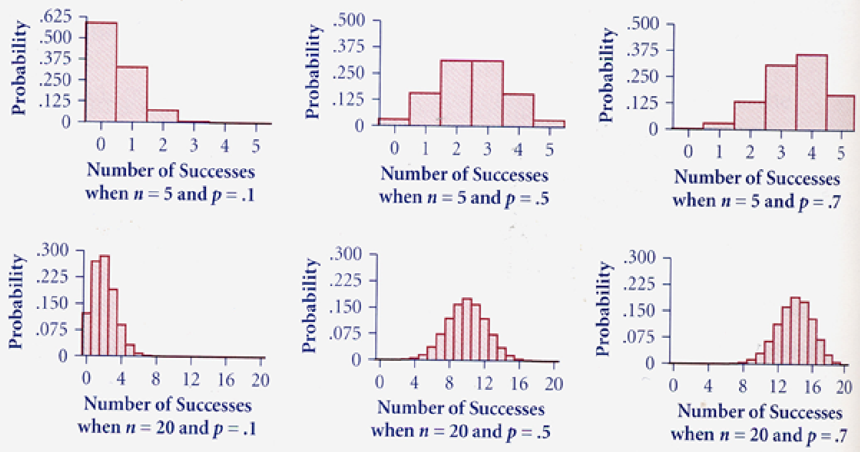
What is the probability of seeing at least 10 of the 16 households with incomes under \$39,000 annually?

$$\binom{16}{10}(.5)^{10}(1-.5)^{16-10} + \ldots + \binom{16}{16}(.5)^{16}(1-.5)^{16-16}$$
$$1 - \text{binomcdf}(16, .5, 9) \approx .227$$

Suppose in a sample of 16 US households, none had incomes below \$39,000. What might you suspect about this sample?

prob. not random

$$P(X=0) = \binom{16}{0}(.5)^0(.5)^{16}$$
$$.000015$$



Display 7.7 Binomial distributions for sample sizes n and different probabilities p.

- When n becomes very large, these formulas become cumbersome, even with a calculator.
- Lucky for us, as n increases, the binomial distribuon becomes more and more Normal.
- So when n is very large, we can use the Normal probability calculaon to approximate the binomial. We will STILL use the binomial formulas for mean and standard deviaon though.

So when IS *n* large enough?

As a rule of thumb, *n* is large enough to use the Normal approximaon when:

$n \cdot p$ <u>and</u> $n \cdot (1-p)$

≥ 10

This is Friday's HW:

HW: p. 523 #19, 21-23, 27ac, 29, 31, 34, 38, 40