$\boxed{8.1}$ Sequence & Series

Pg. 535 5-45 EOO, 53-63 EOO, 65

<u>Sequence</u> - function with a set of natural numbers (positive integers) as its domain

$a_n = 2n$ n = term in the sequence

$a_1 = 2(1)$

$= 2$ 2 is the first term in the sequence

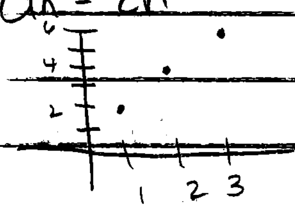
$a_2 = 2(2)$

$= 4$ 4 is the second term in the seq.

$a_n = 2n \Rightarrow$ General term a_n

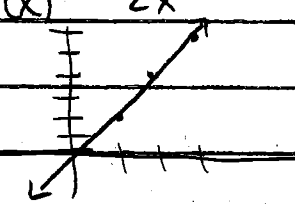
<u>Graphically</u> - sequences are discontinuous

$a_n = 2n$



VS

$f(x) = 2x$



Example! Find the first four terms of the sequence

$\boxed{\text{Explicit}}$

Ⓐ $a_n = (-2)^n n$

$a_1 = (-2)^1(1) = -2$

$a_2 = (-2)^2(2) = 8$

$a_3 = (-2)^3(3) = -24$

$a_4 = (-2)^4(4) = 64$

$\{\ \}$ notation

$\boxed{\{-2, 8, -24, 64\}}$

$\boxed{\text{Recursive}}$ for $n > 1$

Ⓑ $a_1 = -1 \quad a_n = a_{n-1} - 4$

$a_1 = -1$

$a_2 = a_1 - 4 = -1 - 4 = -5$

$a_3 = a_2 - 4 = -5 - 4 = -9$

$a_4 = a_3 - 4 = -9 - 4 = -13$

$\boxed{\{-1, -5, -9, -13\}}$

finite sequence - sequence with an end, terminates

notation: $a_1, a_2, \dots a_n$

finite sum: $S_n = a_1 + a_2 + \dots a_n$ $\quad \sum_{i=1}^{n} a_i$

infinite sequence - sequence that does not end

notation: $a_1, a_2, a_3 \dots$

infinite sum: $S_n = a_1 + a_2 + \dots$ $\quad \sum_{i=1}^{\infty} a_i$

convergent - a sum of an infinite sequence that approaches a single real number

i.e. $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \quad \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \dots$

$\left(\frac{1}{2}\right)^n$ continues to get smaller approaching $\boxed{1}$

divergent - a sequence that does not converge

i.e. $\sum_{n=1}^{\infty} 2^n \quad 2 + 4 + 8 + 16 + 32 \dots$

2^n continues to get larger. No infinite sum

13-20 13. Finite 14. finite 15. finite 16. finite
17. infinite 18. infinite 19. Finite ($2 \leq n \leq 10$)
20. infinite ($n \geq 3$)

$\boxed{8.1}$ cont'd

Summation Rules

① $\sum_{i=1}^{n} i = 1 + 2 \ldots n = \frac{n(n+1)}{2}$

(i.e) $\sum_{i=1}^{20} i = \frac{20(20+1)}{2} = \boxed{110}$

② $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \ldots n^2 = \frac{n(n+1)(2n+1)}{6}$

(i.e) $\sum_{i=1}^{14} i^2 = \frac{14(14+1)(2 \cdot 14+1)}{6} = \frac{14(15)(29)}{6} = \boxed{1015}$

③ $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \ldots n^3 = \frac{n^2(n+1)^2}{4}$

(i.e) $\sum_{i=1}^{25} i^3 = \frac{25^2(25+1)^2}{4} = \boxed{105625}$

Summation Properties   c = constant value

	Property	Example
①	$\sum_{i=1}^{n} c = nc$	$\sum_{i=1}^{12} 3 = 12 \cdot 3 = \boxed{36}$
②	$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$	$\sum_{i=1}^{20} 5(2n+1) = 5\sum_{i=1}^{20} 2n+1$
③	$\sum_{i=1}^{n} (a_i + b_i)$	$\sum_{i=1}^{27} (n^2 + 3n) = \sum_{i=1}^{27} n^2 + \sum_{i=1}^{27} 3n$
④	$\sum_{i=1}^{n} (a_i - b_i)$	$\sum_{i=1}^{35} (n^3 - 2n^2) = \sum_{i=1}^{35} n^3 - \sum_{i=1}^{35} 2n^2$

Example 2 Find the sum for each series

(A) $\sum_{i=1}^{6} 3i-2$

$a_1=3(1)-2=1$ $a_4=3(4)-2=10$

$a_2=3(2)-2=4$ $a_5=3(5)-2=13$

$a_3=3(3)-2=7$ $a_6=3(6)-2=16$

$\sum_{i=1}^{6} = 1+4+7+10+13+16 = \boxed{51}$

(B) $\sum_{i=1}^{8} 5(2)^i$

option 1

$a_1=5(2)^1=10$ $a_3=5(2)^3=40$ $a_5=5(2)^5=160$ $a_7=5(2)^7=640$

$a_2=5(2)^2=20$ $a_4=5(2)^4=80$ $a_6=5(2)^6=320$ $a_8=5(2)^8=1280$

$\sum = 10+20+40+80+160+320+640+1280 = \boxed{2550}$

option 2

$5\sum_{i=1}^{8} 2^i = 5(2^1+2^2+2^3+2^4+2^5+2^6+2^7+2^8)$

$5(2+4+8+16+32+64+128+256)$

$5(510) = \boxed{2550}$

(C) $\sum_{i=1}^{40} 3i^2 = 3\sum_{i=1}^{40} i^2 \Rightarrow$ Rule: $\sum i^2 = \frac{n(n+1)(2n+1)}{6}$

55-58 use rules

$3\left(\frac{40(40+1)(2\cdot 40+1)}{6}\right) = 3\left(\frac{40\cdot 41\cdot 81}{6}\right) = \boxed{65600}$

(D) $\sum_{i=3}^{7} 6i-1$

$a_3=6(3)-1=17$ $a_6=6(6)-1=35$

$a_4=6(4)-1=23$ $a_7=6(7)-1=41$

$a_5=6(5)-1=29$ $\sum = 17+23+29+35+41 = \boxed{145}$

$\boxed{8.1}$ cont'd

Example 3 Evaluate where $x_1 = -3$ $x_2 = 0$ $x_3 = 3$ $x_4 = 6$ and $x_5 = 9$

$$\sum_{i=1}^{4} (x_i^2 + 2x_i)$$

$$a_1 = x_1^2 + 2x_1 = (-3)^2 + 2(-3) = 9 - 6 = 3$$

$$a_2 = x_2^2 + 2x_2 = 0^2 + 2(0) = 0$$

$$a_3 = x_3^2 + 2x_3 = 3^2 + 2(3) = 9 + 6 = 15$$

$$a_4 = x_4^2 + 2x_4 = 6^2 + 2(6) = 36 + 12 = 48$$

$$\Sigma = 3 + 0 + 15 + 48 = \boxed{66}$$