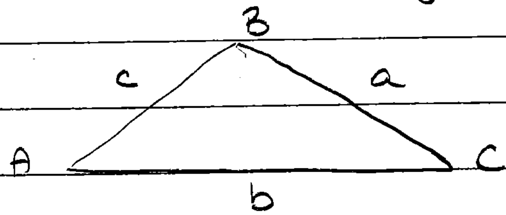
Note cards: Ambiguous Case

8.1 Law of Sines

Calculator: Degree / Radian Mode



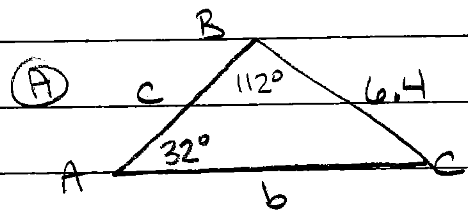
"Solve the Triangle"
Find all 3 angles
and all 3 sides

* can only use pythagorean theorem for right triangles *

Law of Sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Example 1 Solve the triangle

(A)



$\frac{6.4}{\sin 32} = \frac{b}{\sin 112} = \frac{c}{\sin C}$

$32° + 112° + C = 180°$

$144° + C = 180$

$\boxed{C = 36°}$

$\frac{6.4}{\sin 32} = \frac{b}{\sin 112}$

$\frac{b \sin 32}{\sin 32} = \frac{6.4 \sin 112}{\sin 32}$

$\boxed{b = 11.198}$

$\frac{6.4}{\sin 32} = \frac{c}{\sin 36}$

$\frac{c \sin 32}{\sin 32} = \frac{6.4 \sin 36}{\sin 32}$

$\boxed{c = 7.099}$

* Largest angle is always opposite longest side
Smallest " " shortest side

(B) $A = 108° 12'$ $a = 34$ $b = 13$

↳ convert DMS to degrees

$108° 12' = 108.2°$

$$\frac{34}{\sin 108.2} = \frac{13}{\sin B}$$

$$\frac{34\sin B}{34} = \frac{13\sin 108.2}{34}$$

$$B = \sin^{-1}\left(\frac{13\sin 108.2}{34}\right)$$

$$\boxed{B = 21.298°}$$

$108.2 + 21.298 + C = 180$

$129.498 + C = 180$

$\boxed{C = 50.502°}$

$$\frac{c}{\sin 50.502} = \frac{34}{\sin 108.2}$$

$$\frac{c\sin 108.2}{\sin 108.2} = \frac{34\sin 50.502}{\sin 108.2}$$

$$\boxed{c = 27.618}$$

One Solution AAS, ASA

(pg. 590) Ambiguous Case SSA

Given two sides and a non-included angle: a, b and A

$(h = b\sin A)$

Given Angle A is:

① obtuse
- $a \le b$: no solution
- $a > b$: one

② acute
- $a > b$: one
- $a < b$:
  - $a = b\sin A$: One
  - * $a > b\sin A$: Two
  - $a < b\sin A$: none

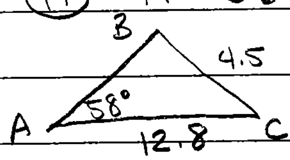
8.1 cont'd

Ambiguous Case: Given a, b and A

SSA → A is acute → $a > b\sin A$

Example 2 Solve the triangle, if possible. Find both solutions if two exist.

(A) $A = 58°$ $a = 4.5$ $b = 12.8$



SSA → $58°$ is acute → $4.5 < 12.8$

$4.5 \overset{?}{} 12.8\sin 58$

$4.5 < 10.855$ $\boxed{\text{no solution}}$

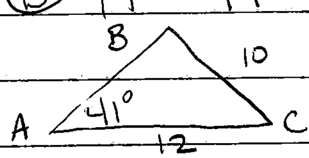
Calculator test: $\dfrac{4.5}{\sin 58} = \dfrac{12.8}{\sin B}$

$$\frac{4.5\sin B}{4.5} = \frac{12.8\sin 58}{4.5}$$

$$B = \sin^{-1}\left(\frac{12.8\sin 58}{4.5}\right)$$

= Domain Error

(B) $A = 41°$ $a = 10$ $b = 12$



SSA → $41°$ is acute → $10 < 12$

$10 \overset{?}{} 12\sin 41$

$10 > 7.873$ Two solutions

* B is the only part of $\triangle ABC$ you can find

Case I

$$\frac{10}{\sin 41} = \frac{12}{\sin B}$$

$$\frac{10\sin B}{10} = \frac{12\sin 41}{10}$$

$$B = \sin^{-1}\left(\frac{12\sin 41}{10}\right)$$

$$\boxed{B = 51.931}$$

$$41 + 51.931 + C = 180$$

$$92.931 + C = 180$$

$$\boxed{C = 87.069}$$

$$\frac{10}{\sin 41} = \frac{c}{\sin 87.069}$$

$$\frac{c\sin 41}{\sin 41} = \frac{10\sin 87.069}{\sin 41}$$

$$\boxed{c = 15.223}$$

Case II

Begin with the supplement of the first angle found

$$B' = 180 - 51.931$$

$$\boxed{B' = 128.069}$$

$$C' + 128.069 + 41 = 180$$

$$C' + 169.069 = 180$$

$$\boxed{C' = 10.931}$$

$$\frac{10}{\sin 41} = \frac{c'}{\sin 10.931}$$

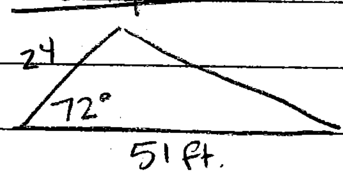
$$\frac{c'\sin 41}{\sin 41} = \frac{10\sin 10.931}{\sin 41}$$

$$\boxed{c' = 2.890}$$

Area of a Triangle SAS

$A = \frac{1}{2}ab\sin C \quad A = \frac{1}{2}ac\sin B \quad A = \frac{1}{2}bc\sin A$

Example 2 Find the area of $\triangle ABC$



$$A = \frac{1}{2}(24)(51)\sin 72^\circ$$

$$= \boxed{582.047 \text{ ft}^2}$$

$\boxed{8.1}$ Law of Sines (Cont'd)

Graphic on back first

Example 3 Find values for b such that the triangle has a) one solution b) two solutions c) no solutions

$A = 42^\circ$ $a = 31$ A is acute

a) one solution

$\boxed{b \le 31}$ or $\frac{b\sin 42}{\sin 42} = \frac{31}{\sin 42}$

$\boxed{b = \frac{31}{\sin 42}}$

Rounding

b) two solutions

$b > 31$ $\frac{b\sin 42^\circ}{\sin 42} < \frac{31}{\sin 42} \approx 46.329$

$31 < b < \frac{31}{\sin 42}$

c) no solutions

$b > 31$ $b\sin 42 > 31$

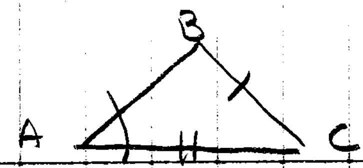
$b > \frac{31}{\sin 42}$

$\boxed{b > \frac{31}{\sin 42^\circ}}$

Applications

18 Law of Sines - Ambiguous Case

SSA Given two sides a, b and non-included angle A, if:



$\angle A$	a to b	a to $b\sin A$	# of solutions
① Obtuse	$a \le b$		none
	$a > b$		one
② * Acute	$a \ge b$		one
	* $a < b$	$a < b\sin A$	none
		$a = b\sin A$	one (Right $\triangle$)
		* $a > b\sin A$	* two Ambiguous Case