Open

Find the det(A)

$A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = 10 - 3 = 7$

7.6 Solutions of Linear Systems (2×2)

Inverse of a 2×2 Matrix

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\det A \neq 0$, then $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Steps to finding the inverse:

1. find $\det A$
2. find $\frac{1}{\det A}$
3. use scalar multiplication w/ $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Example 1: Find the inverse

(A) $A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$2 - 3 = -1$

$\frac{1}{-1} = -1 \quad \begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

(B) $A = \begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$

$\det A = 4 - 2 = 2$

$\det A = 2$

$\frac{1}{2} \begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -1 & 2 \end{bmatrix} = A^{-1}$

(C) $A = \begin{bmatrix} 4 & 4 \\ 2 & 3 \end{bmatrix}$

$12 - 8 = 4$

$\frac{1}{4}\begin{bmatrix} 3 & -4 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 3/4 & -1 \\ -1/2 & 1 \end{bmatrix} = A^{-1}$

(D) $A = \begin{bmatrix} 5 & 3 \\ 10 & 6 \end{bmatrix}$

$30 - 30$

$\det A = 0$

$\frac{1}{0}$ = DNE

Identity Matrix

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2×2

* only with square matrices ie. 2x2, 3x3, 4x4

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 3×3

Example 2: Determine whether A and B are inverses by calculating AB and BA

(A) $A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$BA = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

inverses

(B) $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 7 \end{bmatrix}$ not inverse

$BA = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 0 & 1 \end{bmatrix}$

(C) $A = \begin{bmatrix} 5 & 3 \\ -3 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 \\ -3 & -5 \end{bmatrix}$

$AB = \begin{bmatrix} 5 & 3 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -3 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓ inverse

$BA = \begin{bmatrix} 2 & 3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

Example 3: Use matrix inverse to solve for the systems

(7) or (36)

(35) I comes out pretty

$2x - y = -8$
$3x + y = -2$ $= \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ -2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\begin{bmatrix} -8 \\ -2 \end{bmatrix}$

$A^{-1} = \frac{1}{2+3} = \frac{1}{5}\begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -3/5 & 2/5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -3/5 & 2/5 \end{bmatrix}\begin{bmatrix} -8 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

$2\times2 \quad 2\times1$

2×1

$-\frac{8}{5} + -\frac{2}{5} = -\frac{10}{5}$

$\frac{24}{5} + -\frac{4}{5} = \frac{20}{5}$