7.5 Determinants

Determinant of a 2×2 Matrix A, where

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det A = ad - bc$$

example 1 - Find $\det A$, given the matrix

(A) $A = \begin{bmatrix} -3 & 4 \\ 6 & 8 \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\det A = ad - bc$

$= (-3)(8) - (4)(6)$

$= -24 - 24$

$= \boxed{-48}$

(B) $A = \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$

$= (2)(6) - (1)(5)$

$= 12 - 5$

$= \boxed{7}$

(C) $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$

$= (2)(1) - (-1)(3)$

$= 2 + 3$

$= \boxed{5}$

Example 2: Solve each determinant equation for x

(A) $\det\begin{bmatrix} 5 & x \\ -3 & 2 \end{bmatrix} = 13$

$10 + 3x = 13$

$3x = 3$

$x = 1$

(B) $\det\begin{bmatrix} 3 & 2 \\ x & x \end{bmatrix} = 0$

$3x - 2x = 0$

$x = 0$

(C) $\det\begin{bmatrix} x & 3 \\ x & x \end{bmatrix} = 4$

$x^2 - 3x = 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1)$

$x = 4$

$x = 1$

(D) $\det\begin{bmatrix} x & x \\ 3 & x \end{bmatrix} = 2$

$x^2 - 3x = 2$

$x^2 - 3x - 2 = 0$

$(x-2)(x-1)$

$x = 2$

$x = 1$