7.3 solutions by Row transformations pg. 465 11-21 odd, 33-45 EOD, 65

Example 1: write the augmented matrix for each system.

(A) $3x + 5y = -13$
$2x + 3y = -9$ → $\left[\begin{array}{cc|c} 3 & 5 & -13 \\ 2 & 3 & -9 \end{array}\right]$

(B) $2x + 7y = 1$
$5x + y = -15$ → $\left[\begin{array}{cc|c} 2 & 7 & 1 \\ 5 & y & -15 \end{array}\right]$

(C) $4x - 2y + 3z = 4$
$3x + 5y + z = 7$ → $\left[\begin{array}{ccc|c} 4 & -2 & 3 & 4 \\ 3 & 5 & 1 & 7 \\ 5 & -1 & 4 & 7 \end{array}\right]$
$5x - y + 4z = 7$

Example 2: write the system of equations associated with each augmented matrix.

(A) $\left[\begin{array}{cc|c} 1 & -5 & -18 \\ 6 & 2 & 20 \end{array}\right]$ → $x - 5y = -18$
$6x + 2y = 20$

(B) $\left[\begin{array}{cc|c} 2 & 5 & 5 \\ -1 & 0 & 3 \end{array}\right]$ → $2x + 5y = 5$
$-x = 3$

(C) $\left[\begin{array}{ccc|c} 1 & 3 & 5 & 1 \\ 2 & 6 & 1 & 3 \\ 5 & -1 & 0 & -2 \end{array}\right]$ → $x + 3y + 5z = 1$
$2x + 6y + z = 3$
$5x - 6y = -2$

(D) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$ → $x + z = 4$
$y = 2$
$z = 3$

w/ (D) → plug z in and solve for x
$x = 1$

Row Echelon Method

Matrix row transformations are used to transform the augmented matrix of a system into one that is in echelon

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 7 \\ 0 & 1 & 2 & 9 \\ 0 & 0 & 1 & 4 \end{array}\right] \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

* use row echelon method
* write system of equations
* solve for variables, use back substitution.

Example 3: solve each equation using row operations.

(A) $x + 2y = 5$ $\rightarrow$ $\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 2 & 1 & -2 \end{array}\right]$

$2x + y = -2$

$-2R1 + R2$

$$\begin{array}{rrr|r} & -2 & -4 & -10 \\ + & 2 & 1 & -2 \\ \hline & 0 & -3 & -12 \end{array} \rightarrow \left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & -3 & -12 \end{array}\right]$$

$\frac{R2}{3}$ $\left[\frac{0}{-3} \quad \frac{-3}{-3} \middle| \frac{-12}{-3}\right]$ $\rightarrow$ $\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & 1 & 4 \end{array}\right]$

$x + 2y = 5$

$y = 4$

substite y $\rightarrow$ $x + 2(4) = 5$

$x + 8 = 5$

$x = -3$

solution: $\boxed{(-3, 4)}$

(B) $3x - 2y = 4$ $\rightarrow$ $\left[\begin{array}{cc|c} 3 & -2 & 4 \\ 3 & 1 & -2 \end{array}\right]$
$3x + y = -2$

$-1R1 + R2$

$$\begin{array}{rr|r} -3 & 2 & -4 \\ + \ 3 & 1 & -2 \\ \hline 0 & 3 & -6 \end{array} \rightarrow \left[\begin{array}{cc|c} 3 & -2 & 4 \\ 0 & 3 & -6 \end{array}\right]$$

$\frac{R2}{3}$

$$\begin{array}{rr|r} 0 & 3 & -6 \\ 0 & 1 & -2 \end{array} \rightarrow \left[\begin{array}{cc|c} 3 & -2 & 4 \\ 0 & 1 & -2 \end{array}\right]$$

$3x - 2y = 4$ $\rightarrow$ substitute y & solve
$y = -2$

$$3x - 2(-2) = 4$$
$$3x + 4 = 4$$
$$3x = 0$$
$$x = 0$$

solution: $(0, -2)$

(C) $4x + y = 5$ $\rightarrow$ $\left[\begin{array}{cc|c} 4 & 1 & 5 \\ 2 & 1 & 3 \end{array}\right]$
$2x + y = 3$

$-1R1 + 2R2$

$$\begin{array}{rr|r} -4 & -1 & -5 \\ 4 & 2 & 6 \\ \hline 0 & 1 & 1 \end{array} \rightarrow \left[\begin{array}{cc|c} 4 & 1 & 5 \\ 0 & 1 & 1 \end{array}\right]$$

$4x + y = 5$
$y = 1$ $\rightarrow$ substitute y & solve

$$4x + 1 = 5$$
$$4x = 4$$
$$x = 1$$

solution: $(1, 1)$

7.3 cont'd

Open

use row operations to solve the system.

$$\begin{aligned} x - y &= 1 \\ -x + y &= -1 \end{aligned} \longrightarrow \left[\begin{array}{cc|c} 1 & -1 & 1 \\ -1 & 1 & -1 \end{array}\right]$$

$$R1 + R2 \rightarrow \quad 0 \;\; 0 \,|\, 0 \qquad \left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$x - y = 1$ infinite solutions

→ graph both lines to show they are the same.

Example 4: use row operations to solve the system.

(A)

$$\begin{aligned} x - z &= -3 \\ y + z &= 9 \\ x + z &= 7 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 1 & 9 \\ 1 & 0 & 1 & 7 \end{array}\right]$$

$$-1R1 + R3 = \begin{array}{r} -1 \;\; 0 \;\; 1 \;|\; 3 \\ +\; 1 \;\; 0 \;\; 1 \;|\; 7 \\ \hline 0 \;\; 0 \;\; 2 \;|\; 10 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 2 & 10 \end{array}\right]$$

$$\frac{R3}{2} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 1 & 5 \end{array}\right] \rightarrow \begin{aligned} x - y &= 3 \\ y + z &= 7 \\ z &= 5 \end{aligned}$$

substitute z & solve $\quad y + (5) = 7$

$$y = 2$$

$$x - (2) = 3$$

$$x = 5$$

solution: $(5, 2, 5)$

(B.) $x + y = 1$
$2x - z = 0$
$y + 2z = -2$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 2 & 0 & -1 & 0 \\ 0 & 1 & 2 & -2 \end{array}\right]$

$-2R1 + R2 \rightarrow$
$$\begin{array}{ccc|c} -2 & -2 & 0 & -2 \\ 2 & 0 & -1 & 0 \\ \hline 0 & -2 & -1 & -2 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -2 & -1 & -2 \\ 0 & 1 & 2 & -2 \end{array}\right]$$

$2R3 + R2 \rightarrow$
$$\begin{array}{ccc|c} 0 & 2 & 4 & -4 \\ 0 & -2 & -1 & -2 \\ \hline 0 & 0 & 3 & -6 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -2 & -1 & -2 \\ 0 & 0 & 3 & -6 \end{array}\right]$$

$x + y = 1$
$-2y - z = -2$
$3z = 6$

$3z = 6$
$z = 2$

substitute z in & solve

$-2y - 2 = -2$
$-2y = 0$
$y = 0$

$x + 0 = 1$
$x = 1$

solution: $(1, 0, 2)$

(c) $x + 3y - 6z = 7$
$2x - y + 2z = 0$
$x + y + 2z = -1$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 2 & -1 & 2 & 0 \\ 1 & 1 & 2 & -1 \end{array}\right]$$

$-2(R3) + R2 =$
$$\begin{array}{ccc|c} -2 & -2 & -4 & 2 \\ 2 & -1 & 2 & 0 \\ \hline 0 & -3 & -2 & 2 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 2 & -1 & 2 & 0 \\ 0 & -3 & -2 & 2 \end{array}\right]$$

$-2(R1) + R2 =$
$$\begin{array}{ccc|c} -2 & -6 & 12 & -14 \\ 2 & -1 & 2 & 0 \\ \hline 0 & -7 & 14 & -14 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 0 & -7 & 14 & -14 \\ 0 & -3 & -2 & 2 \end{array}\right]$$

$$\frac{(R2)}{-7} = \begin{array}{ccc|c} 0 & 1 & -2 & 2 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 0 & 1 & -2 & 2 \\ 0 & -3 & -2 & 2 \end{array}\right]$$

$3(R2) + R3 =$
$$\begin{array}{ccc|c} 0 & 3 & -6 & 6 \\ 0 & -3 & -2 & 2 \\ \hline 0 & 0 & -8 & 8 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & -8 & 8 \end{array}\right]$$

stop & solve: $x + 3y - 6z = 7$
$y - 2z = 2$
$-8z = 8$

$-8z = 8$
$z = -1$

$y - 2(-1) = 2$
$y + 2 = 2$
$y = 0$

$x + 0 + 6 = 7$
$x = 1$

solution: $(1, 0, -1)$

keep going for row echelon form

$$\frac{(R3)}{-8} = \begin{array}{ccc|c} 0 & 0 & 1 & -1 \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$x + 3y - 6z = 7$
$-y - 2z = 2$
$z = -1$ $\rightarrow$ same steps to solve