hw pg 454 5-11 odd, 25, 27, 33, 37

7.2 Solutions in Three Variables

Open - How can you verify (-3, 6, 1) is a solution to the following system?

$2x + y - z = -1$
$x - y + 3z = -6$
$-4x + y + z = 19$

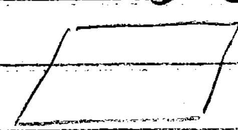
Substitute (-3, 6, 1) for (x, y, z) in ALL equations to verify

$2(-3) + 6 - 1 = -1$ $-3 - 6 + 3(1) = -6$ $-4(-3) + 6 + 1 = 19$
$-1 = -1$ ✓ $-6 = -6$ ✓ $19 = 19$ ✓

Linear Equations in Three Dimensions

$Ax + By + Cz = D$

Graph: plane in 3-dimensions



Intersection of planes (pg. 449 - graphic)

solution may be:

1. (x, y, z) ⇒ point
2. dependent ⇒ line; infinite set of (x, y, z)
3. independent ⇒ parallel planes; empty set

Solving systems in three variables

1. use elimination to eliminate the same variable from two equations
2. Create a new system with those equations
3. Solve and substitute

Example 1 Solve analytically

⑧ $2x + y + z = 9$ ①
$-x - y + z = 1$ ②
$3x - y + z = 9$ ③

Eliminate y from ② & ③ using ①

$$\begin{array}{r} 2x + y + z = 9 \\ -x - y + z = 1 \\ \hline x + 2z = 10 \end{array} \qquad \begin{array}{r} 2x + y + z = 9 \\ 3x - y + z = 9 \\ \hline 5x + 2z = 9 \end{array}$$

Create a new system $x + 2z = 10$, $5x + 2z = 18$ — Use elimination or subst. to solve

Elimination

$$\begin{array}{r} x + 2z = 10 \\ -1(5x + 2z = 18) \end{array} \Rightarrow \begin{array}{r} x + 2z = 10 \\ -5x - 2z = -18 \\ \hline -4x = -8 \\ \boxed{x = 2} \end{array}$$

Substitute $x = 2$

$x + 2z = 10$
$2 + 2z = 10$
$2z = 8$
$\boxed{z = 4}$

or Substitution

$x + 2z = 10 \Rightarrow x = \boxed{-2z + 10}$
$5\boxed{x} + 2z = 18$

$5(-2z + 10) + 2z = 18$
$-10z + 50 + 2z = 18$
$-8z = -32$
$\boxed{z = 4}$

Substitute $z = 4$

$x + 2z = 10$
$x + 2(4) = 10$
$x + 8 = 10$
$\boxed{x = 2}$

Substitute x and z in any of the original questions to find y

$2x + y + z = 9$
$2(2) + y + 4 = 9$
$y + 8 = 9$
$\boxed{y = 1}$

Solution: $\boxed{(2, 1, 4)}$

7.2 cont'd

$(2,1,4) = (x,y,z)$

$2x + y + z = 9$ $-x - y + z = 1$ $3x - y + z = 9$

$2(2) + 1 + 4 = 9$ $-2 - 4 + 4 = 1$ $3(2) - 1 + 4 = 9$

$9 = 9$ ✓ $1 = 1$ ✓ $9 = 9$ ✓

12
$$\left.\begin{aligned} 3x - 2y - 8z &= 1 \\ 9x - 6y - 24z &= -2 \\ x - y + z &= 1 \end{aligned}\right\}$$

Eliminate any variable
I will choose to eliminate z from 1 : 2 using 3

$$\begin{aligned} 3x - 2y - 8z &= 1 \\ 8(x - y + z &= 1) \end{aligned} \Rightarrow \begin{aligned} 3x - 2y - 8z &= 1 \\ 8x - 8y + 8z &= 8 \\ \hline 11x - 10y &= 9 \end{aligned}$$

$$\begin{aligned} 9x - 6y - 24z &= -2 \\ 24(x - y + z &= 1) \end{aligned} \Rightarrow \begin{aligned} 9x - 6y - 24z &= -2 \\ 24x - 24y + 24z &= 24 \\ \hline 33x - 30y &= 22 \end{aligned}$$

New system

$$\begin{aligned} 11x - 10y &= 9 \\ 33x - 30y &= 22 \end{aligned}$$

Elimination

$$\Rightarrow \begin{aligned} -3(11x - 10y &= 9) \\ 33x - 30y &= 22 \end{aligned} \Rightarrow \begin{aligned} -33x + 30y &= -27 \\ 33x - 30y &= 22 \\ \hline 0 &\neq -5 \end{aligned}$$

$\boxed{\text{no solution}}$

Stop here!

(26) $2x + y - z = -4$ ①
$y + 2z = 12$ ②
$2x - z = -4$ ③

Reduce down to two variables in two equations
Multiple ways to solve!

Eliminate x from ① using ③

$$2x + y - z = 4$$
$$-1(2x - z = -4) \rightarrow$$

$$2x + y - z = -4$$
$$-2x + z = 4$$
$$\boxed{y = 0}$$

Substitute $y=0$ into

$$y + 2z = 12$$
$$0 + 2z = 12$$
$$\boxed{z = 6}$$

Substitute $y=0$ $z=6$ into ①

$$2x + y - z = -4$$
$$2x + 0 - 6 = -4$$
$$2x - 6 = -4$$
$$2x = 2$$
$$\boxed{x = 1}$$

Solution: $\boxed{(1, 0, 6)}$