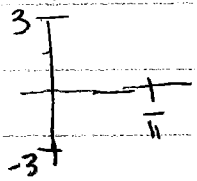
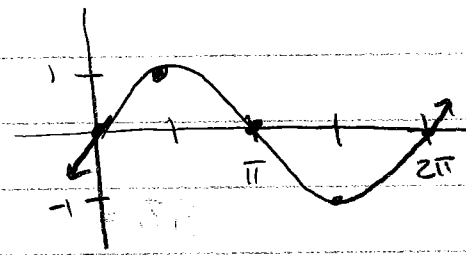


6.4 Graphs of Sine & Cosine

Sine

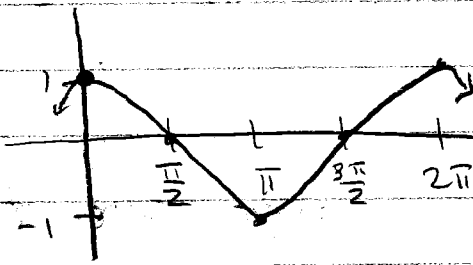
* Scale graphs
 $\pi \approx 3$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin \theta$	0	1	0	-1	0



Cosine

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos \theta$	1	0	-1	0	1

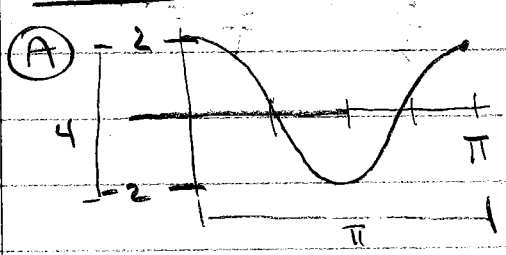


Period - 'time' needed to complete one cycle

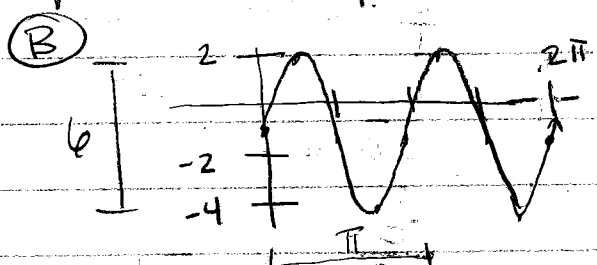
$\left. \begin{array}{l} \sin/\cos \\ \csc/\sec \end{array} \right\} 2\pi$

Amplitude - $\frac{1}{2}$ distance between the max and min

Example: State the amplitude and period



Amp = 2
Period = π



Period = π
Amp = 3

$\sin \theta / \cos \theta$ Equations

$f(x) = a \cos b\theta$

$f(x) = a \sin b\theta$

Amplitude = a (always positive)

Period = $\frac{2\pi}{b}$

Example 2 Find the period and amplitude

(A) $y = -\cos \frac{2}{3}x$

Amplitude = 1

$$\begin{aligned} \text{Period} &= \frac{2\pi}{\frac{2}{3}} \\ &= 3\pi \end{aligned}$$

(B) $y = \frac{1}{2} \sin \frac{x}{4}$
 $= \frac{1}{2} \sin \frac{1}{4}x$

Amplitude = $\frac{1}{2}$

$$\begin{aligned} \text{Period} &= \frac{2\pi}{\frac{1}{4}} \\ &= 8\pi \end{aligned}$$

Reflections

$f(-x) \Rightarrow$ Reflection in y-axis

$-f(x) \Rightarrow$ Reflection in x-axis

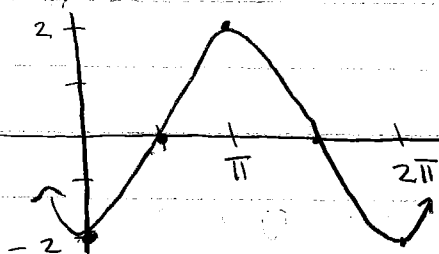
* Does not change amplitude

$$f(x) = -2 \cos x$$

Reflection
in x-axis

Amp = 2

Period = $\frac{2\pi}{1}$
 $= 2\pi$



(6.4) cont'd

Shifts

$$f(x) = a \cos b(x-c) + d$$

$$f(x) = a \sin b(x-c) + d$$

Phase Shift (c) horizontal shift

① $f(x) = \cos(x - \pi)$

Right π

② $f(x) = \sin(2x + \pi)$

$$= -\sin 2\left(x + \frac{\pi}{2}\right)$$

Left $\frac{\pi}{2}$

$$\text{Period} = \frac{2\pi}{2}$$
$$= \pi$$

* factor 2 out

Vertical shift (d)

① $f(x) = \cos x - 2$

down 2

② $f(x) = 3 + \sin x$

up 3

Example 3 Write the equation for the function described by the given characteristics

Ⓐ cosine curve with a period of $\frac{\pi}{2}$, amplitude of 3, phase shift π , up 2, reflected in x-axis

$$\text{Period} = \frac{2\pi}{b}$$

$$\frac{\pi}{2} = \frac{2\pi}{b}$$

$$4\pi = b\pi \quad b = 4$$

$$y = -3 \cos 4(x - \pi) + 2$$

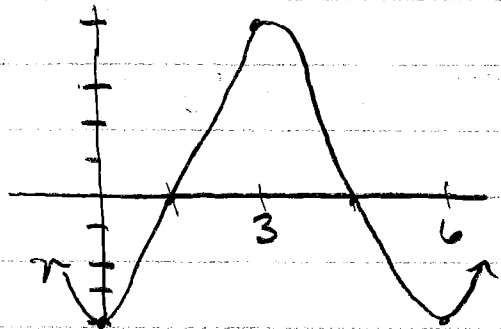
Example 4 Sketch the graph of the function.
Include one full period.

(A) $y = -4 \cos \frac{\pi x}{3}$
 $= -4 \cos \frac{\pi}{3} x$

(-) Reflection in x

4 Amplitude = 4

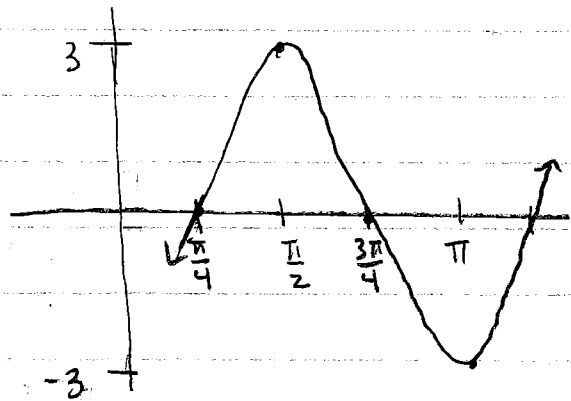
$\frac{\pi}{3}$ Period = $\frac{2\pi}{\frac{\pi}{3}}$
 $= 6$



(B) $y = 3 \sin(2x - \frac{\pi}{4})$
 $= 3 \sin 2(x - \frac{\pi}{4})$

3 amplitude
 $(-\frac{\pi}{4})$ Right $\frac{\pi}{4}$

2 Period = $\frac{2\pi}{2}$
 $= \pi$



(C) $y = \cos(x - \pi) + 1$
 Period = 2π

(-) Reflection in y-axis

$(-\pi)$ Right π

(+1) up 1

