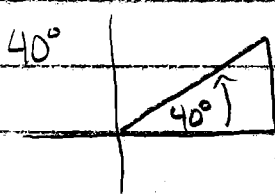
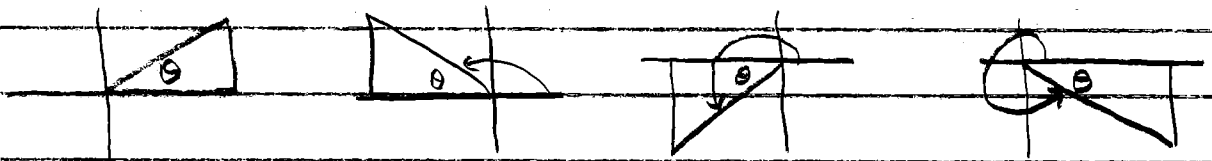
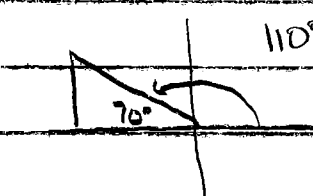


6.3 Trigonometric Functions of Any Angle

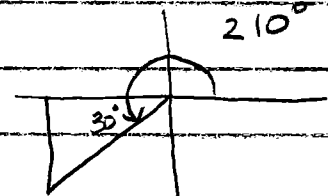
Reference Angle - positive acute angle θ formed by the terminal side of an angle and the horizontal axis



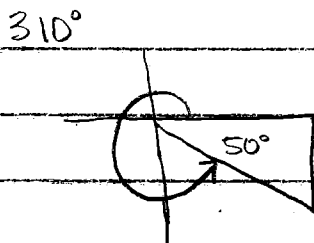
$$\text{Ref } \angle = 40^\circ$$



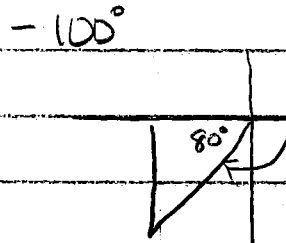
$$\text{Ref } \angle = 70^\circ$$



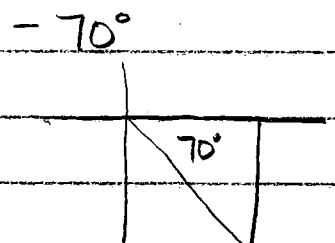
$$\text{Ref } \angle = 30^\circ$$



$$\text{Ref } \angle = 50^\circ$$

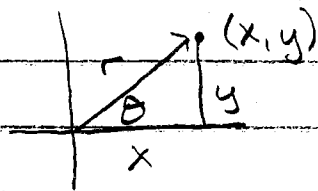


$$\text{Ref } \angle = 80^\circ$$



$$\text{Ref } \angle = 70^\circ$$

Trig functions



$$\sin \theta = \frac{y}{r}$$

$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}$$

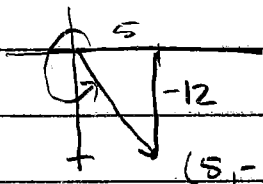
$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$

* r is always positive

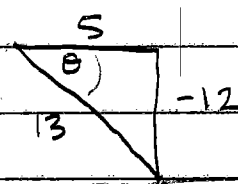
Example 1 Let $(5, -12)$ be a point on the terminal side of θ . Find $\sin\theta$, $\cos\theta$ and $\tan\theta$.



$$r^2 = 5^2 + (-12)^2$$

$$r^2 = 169$$

$$r = 13$$



$\sin\theta = \frac{-12}{13}$	$\cos\theta = \frac{5}{13}$	$\tan\theta = \frac{-12}{5}$
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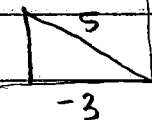
Quadrants

II (-, +)	I (+, +)	
(S) $\sin\theta = +$ $\cos\theta = -$ $\tan\theta = -$	$\sin\theta = +$ (A) $\cos\theta = +$ $\tan\theta = +$	* $\csc\theta$, $\sec\theta$, $\tan\theta$ are +/-
$\sin\theta = -$ $\cos\theta = -$ (T) $\tan\theta = +$	$\sin\theta = -$ $\cos\theta = +$ $\tan\theta = -$ (C)	In the same quadrants
III (-, -)	IV (+, -)	* Calculator $\csc\theta = \frac{1}{\sin\theta}$

Example 2

(A) Given $\cos\theta = -\frac{3}{5}$ and $\sin\theta > 0$, find $\sin\theta$, $\tan\theta$ and $\cot\theta$

$\cos\theta (-)$ } Quad II
 $\sin\theta (+)$ }



$$r^2 = x^2 + y^2$$

$$5^2 = (-3)^2 + y^2$$

$$25 = 9 + y^2$$

$$16 = y^2$$

$$4 = y$$

$$\sin\theta = \frac{4}{5}$$

$$\tan\theta = \frac{4}{-3}$$

$$\cot\theta = -\frac{3}{4}$$

$$\sin\theta = \frac{y}{r}$$

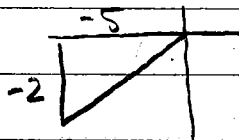
$$\tan\theta = \frac{y}{x}$$

$$\cot\theta = \frac{x}{y}$$

16.3 cont'd

Ⓑ $\tan \theta = \frac{2}{5}$ and $\cos \theta < 0$, find $\sin \theta$ and $\sec \theta$

$\tan \theta (+)$
 $\cos \theta (-)$ } Quad III



$$r^2 = (-5)^2 + (-2)^2$$

$$r^2 = 25 + 4$$

$$r^2 = 29$$

$$r = \sqrt{29}$$

$$\sin \theta = \frac{-2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}}$$

$$\boxed{\sin \theta = -\frac{2\sqrt{29}}{29}}$$

$$\sin \theta = \frac{y}{r}$$

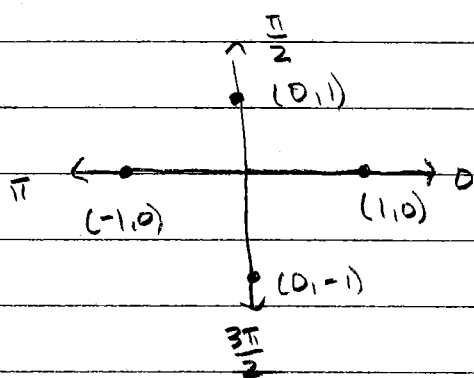
$$\sec \theta = \frac{r}{x}$$

$$\boxed{\sec \theta = -\frac{\sqrt{29}}{5}}$$

Quadrant Angles

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$0, 90^\circ, 180^\circ, 270^\circ, 360^\circ$



$$r = 1$$

$$\sin \theta = \frac{y}{r}$$

$$\sin 0 = \frac{0}{1} = 0$$

$$\sin \frac{\pi}{2} = \frac{1}{1} = 1$$

$$\sin \pi = \frac{0}{1} = 0$$

$$\sin \frac{3\pi}{2} = \frac{-1}{1} = -1$$

$$\cos \theta = \frac{x}{r}$$

$$\cos 0 = \frac{1}{1} = 1$$

$$\cos \frac{\pi}{2} = \frac{0}{1} = 0$$

$$\cos \pi = \frac{-1}{1} = -1$$

$$\cos \frac{3\pi}{2} = \frac{0}{1} = 0$$

$$\tan \theta = \frac{y}{x}$$

$$\tan 0 = \frac{0}{1} = 0$$

$$\tan \frac{\pi}{2} = \frac{1}{0} \text{ undefined}$$

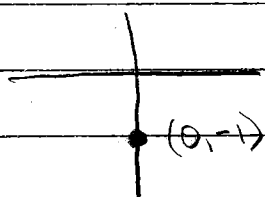
$$\tan \pi = \frac{0}{-1} = 0$$

$$\tan \frac{3\pi}{2} = \frac{-1}{0} \text{ undefined}$$

Example 3 Find the six trig functions given the constraints: $\tan \theta$ is undefined
 $\pi < \theta < 2\pi$

$\tan \theta = \frac{y}{x} \Rightarrow$ undefined where $x=0$

$\pi < \theta < 2\pi$  $x=0 \in \frac{3\pi}{2}$



$\sin \theta = -1$

$\csc \theta = -1$

$\cos \theta = 0$

$\sec \theta = \text{undefined}$

$\tan \theta = \text{undefined}$

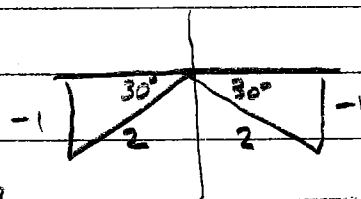
$\cot \theta = 0$

Example 4 Find two solutions: 1) in degrees $0 \leq \theta < 360^\circ$
 2) in radians $0 \leq \theta < 2\pi$

Ⓐ $\sin \theta = -\frac{1}{2}$

Sine is negative in QIII & QIV

$\sin \theta = \frac{y}{r}$



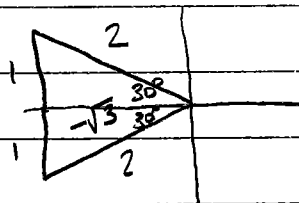
degrees: $\theta = 210^\circ$
 $\theta = 330^\circ$

radians: $\theta = \frac{7\pi}{6}$
 $\theta = \frac{11\pi}{6}$

Ⓑ $\sec \theta = -\frac{2\sqrt{3}}{3}$

sec is negative in QII, QIII

$\sec \theta = \frac{r}{x} \quad \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$



$\frac{r}{x} = \frac{2}{\sqrt{3}}$

degrees: $\theta = 150^\circ$
 $\theta = 210^\circ$

radians: $\theta = \frac{5\pi}{6}$
 $\theta = \frac{7\pi}{6}$