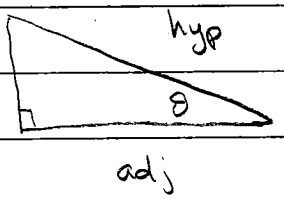
Youtube: Roller Coaster

warm up: rationalize radicals & fractions

6.2 Right Triangle Trig

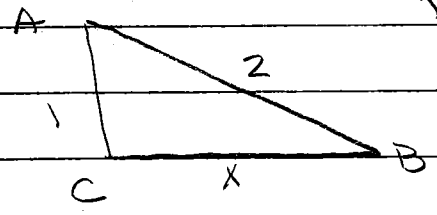
Given θ is an acute angle of a right triangle



$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

$\csc\theta = \frac{hyp}{opp}$ $\sec\theta = \frac{hyp}{adj}$ $\cot\theta = \frac{adj}{opp}$

Example 1 Find the six trig ratios of $\triangle ABC$ for angle A.



$1^2 + x^2 = 2^2$

$1 + x^2 = 4$

$x^2 = 3$

$x = \sqrt{3}$

$\sin A = \frac{\sqrt{3}}{2}$ $\cos A = \frac{1}{2}$ $\tan A = \sqrt{3}$

$\csc A = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\sec A = 2$ $\cot A = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

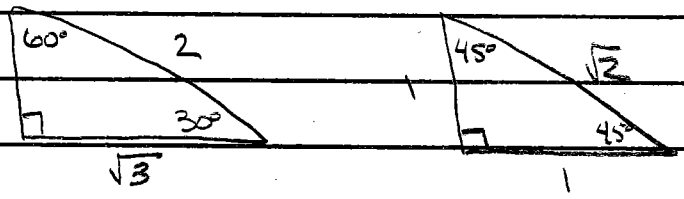
Inverse functions - used to find missing angles

$\cos^{-1}(\cos A) = \cos^{-1}\left(\frac{1}{2}\right)$

$A = 60°$

* Degree mode
Radian mode

Special Triangles



Example 2 Find sin, cos, and tan of each angle $30°, 60°, 45°$ $\left(\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}\right)$

$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$	$\tan 30° = \frac{\sqrt{3}}{3}$
$\sin 60° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$	$\tan 60° = \sqrt{3}$
$\sin 45° = \frac{\sqrt{2}}{2}$	$\cos 45° = \frac{\sqrt{2}}{2}$	$\tan 45° = 1$

Trig Identities

Reciprocal Identities:

$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\tan\theta = \frac{1}{\cot\theta}$
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

* Calculator:

1) $\sec 30° = \frac{1}{\cos 30°}$
$= 1.155$

2) $\cot\frac{\pi}{7} = \frac{1}{\tan\frac{\pi}{7}}$
$= 2.076$

Quotient Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

① $\sin^2\theta + \cos^2\theta = 1$

② $1 + \cot^2\theta = \csc^2\theta$

Proof: $\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$1 + \cot^2\theta = \csc^2\theta$

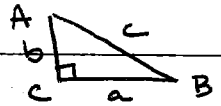
③ $\tan^2\theta + 1 = \sec^2\theta$

Proof: $\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

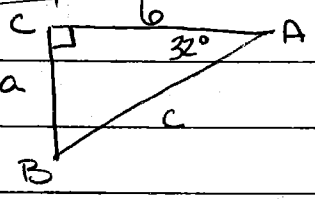
$\tan^2\theta + 1 = \sec^2\theta$

6.2 cont'd (3 decimals)

"Solving a triangle" - find all missing sides and angles



Example 2 Solve ΔABC



(A)

$$\frac{6 \cdot \sin 32}{6} = \frac{a}{6} \cdot 6$$

$$6 \sin 32^\circ = a$$

$$\boxed{3.180 = a}$$

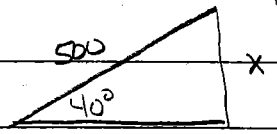
Rounding

$$32 + B + 90 = 180$$

$$122 + B = 180$$

$$\boxed{B = 58^\circ}$$

Example 3 A roller coaster inclines at 40° with the horizontal. If the length of the track is 500ft, how high is the highest point?



$$\sin 40^\circ = \frac{x}{500}$$

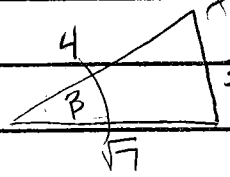
$$500 \sin 40^\circ = x$$

$$\boxed{321.394 \text{ ft} = x}$$

World records

Wikipedia: Magnum XL-200 first complete-circuit roller coaster built over 200ft. (1989)

Example 4 Use trig identities to find the indicated trig functions: $\cos B = \frac{\sqrt{7}}{4}$



a) $\sin B = \frac{3}{4}$

b) $\cos(90 - B) = \frac{3}{4}$

c) $\tan B = \frac{3\sqrt{7}}{7}$

Pythagorean Theorem

(A) $\cos^2 B + \sin^2 B = 1$

$\left(\frac{\sqrt{7}}{4}\right)^2 + \sin^2 B = 1$

$\frac{7}{16} + \sin^2 B = 1$

$\sin^2 B = 1 - \frac{7}{16}$

$\sin^2 B = \frac{9}{16}$

$\sin B = \frac{3}{4}$

(B) $\cos(90 - B) = \sin B$

$= \frac{3}{4}$

(C) $\tan B = \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{3\sqrt{7}}{7}$

Proofs

Example 5 Use Identities to prove (37-46)

more examples

(A) $\cos\theta \sec\theta = 1$

$\cos\theta \cdot \frac{1}{\cos\theta} = 1$

$\frac{\cos\theta}{\cos\theta} = 1$

$1 = 1$

rearrange pythagorean

(B) $\frac{\tan B + \cot B}{\tan B} = \csc^2 B$

$\frac{\tan B}{\tan B} + \frac{\cot B}{\tan B} = \csc^2 B$

$1 + \cot B \cdot \cot B = \csc^2 B$

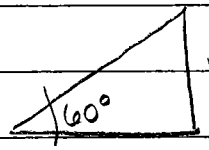
$1 + \cot^2 B = \csc^2 B$

$\csc^2 B = \csc^2 B$

6.2 cont'd

Example 6 Find θ in degrees $(0 < \theta < 90°)$ and radians $(0 < \theta < \frac{\pi}{2})$ without a calc

(A) $\cot\theta = \frac{\sqrt{3}}{3}$

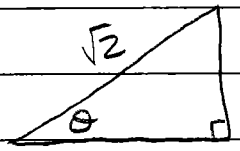


$\frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$\cot = \frac{adj}{opp}$

$60° \quad \frac{\pi}{3}$

(B) $\sec\theta = \sqrt{2}$

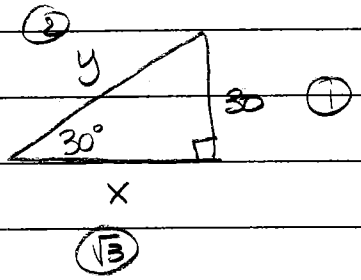


$\frac{hyp}{adj} = \frac{\sqrt{2}}{1}$

$45° \quad \frac{\pi}{4}$

Example 7 Solve for x and y without a calc

(A)



30-60-90

$\frac{1}{30} \cdot \frac{\sqrt{3}}{x} = \frac{2}{y}$

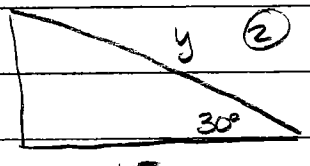
$\frac{1}{30} = \frac{\sqrt{3}}{x}$

$x = 30\sqrt{3}$

$\frac{1}{30} = \frac{2}{y}$

$y = 60$

(B)



$\frac{1}{x} = \frac{\sqrt{3}}{15} = \frac{2}{y}$

$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{15\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$= \frac{15\sqrt{3}}{3}$

$x = 5\sqrt{3}$

$\frac{y\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{\sqrt{3}\sqrt{3}}$

$y = \frac{30\sqrt{3}}{3}$

$y = 10\sqrt{3}$

Applications: