pg. 375 1, 5, 8, 9, 13, 15, 17, 21-37 odd, 43, 47, 49

$\boxed{5.6}$ Further Applications and Modeling

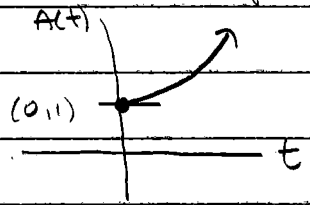
Exponential Growth

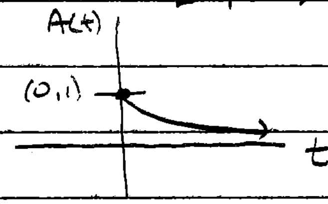
$A(t) = A_0 e^{kt}$

Exponential Decay

$A(t) = A_0 e^{-kt}$

t = time ; restricted to $[0, \infty)$





$A(t)$ = amount present at time t

A_0 = initial quantity at time = 0

k = growth / decay constant rate

half-life - amount of time it takes for a quantity to reach ½ its original amount

Carbon dating - ages of fossils/remains are determined by the amount of carbon 14 present. The half-life of carbon 14 is 5700 years

$$A(t) = A_0 e^{-kt} \quad \text{where } k = \frac{\ln 2}{5700}$$

Example 1 #2

$$A(t) = A_0 e^{-\frac{\ln 2}{5700}t}$$

$$\boxed{A(t)} = A_0 e^{-.0001216t}$$

$$A(t) = \boxed{.60(A_0)}$$

$$.60(\cancel{A_0}) = \cancel{A_0} e^{-.0001216t}$$

$$.60 = e^{-.0001216t}$$

$$\frac{\ln .60}{-.0001216} = \frac{-.0001216t(\ln e)}{-.0001216}$$

$$4200.869 = t$$

$$\boxed{4200.869 \text{ years}}$$

(time for # 1 HW)

⑤ Radioactive lead 210 has ½ life of 21.7 years

ⓐ Find an exponential decay model for lead 210

$$k = \frac{\ln 2}{21.7} = .032$$

$$A(t) = A_0 e^{-.032t}$$

ⓑ How long to decay from 500g to 400g

$A(t) = 400 \quad A_0 = 500$

$$\frac{400}{500} = \frac{500 e^{-.032t}}{500}$$

$$.8 = e^{-.032t}$$

$$\ln .8 = -.032t \ln e$$

$$\frac{\ln .8}{-.032} = \frac{-.032t}{-.032}$$

$$6.973 = t$$

$$\boxed{6.973 \text{ years}}$$

ⓒ $A(t) = 500e^{-.032(10)}$

$$= \boxed{363.075 \text{ grams}}$$

Decibel - measures the loudness of sound

$$d = 10 \log \frac{I}{I_0}$$

I = intensity

I_0 = threshold sound

Example 2 Compare the decibels of Lord of the Rings action sequences ($10^{9.5} I_0$) to a ringing phone (80 decibels)

$$d(LoR) = 10 \log \frac{10^{9.5} \cancel{I_0}}{\cancel{I_0}}$$

$$= 10 \log 10^{9.5}$$

$$= 10(9.5) = 95 \text{ decibels}$$

$$\frac{10^{9.5} I_0}{10^{8.0} I_0} = 10^{1.5} = \boxed{31.6 \text{ times as loud}}$$

$\boxed{5.6}$ cont'd

Compound Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A = Final balance at time, t
P = Principal
r = rate
n = # of compounding/yr
t = time in years

Monthly: $P\left(1+\frac{r}{12}\right)^{12t}$
Quarterly: $P\left(1+\frac{r}{4}\right)^{4t}$
Daily: $P\left(1+\frac{r}{365}\right)^{365t}$

Example 3 (#19)

$$\frac{30{,}000}{27{,}000} = \frac{27{,}000\left(1+\frac{.06}{4}\right)^{4t}}{27{,}000}$$

$$1.111 = (1.015)^{4t}$$

$$\frac{\log 1.111}{4\log 1.015} = \frac{4t \log 1.015}{4\log 1.015}$$

$$1.767 = t$$

$$\boxed{t = 1.8 \text{ years}}$$

(#20) Doubling time:

a) quarterly: $2P = P\left(1+\frac{.025}{4}\right)^{4t}$

$$2 = 1.00625^{4t}$$

$$\ln 2 = \ln 1.00625^{4t}$$

$$\frac{\ln 2}{(4)\ln 1.00625} = \frac{4t \ln 1.00625}{4(\ln 1.00625)}$$

$$\boxed{27.8\text{-}12 \text{ years} = t}$$

b) continuously: $2 = e^{.025t}$

$$\frac{\ln 2}{.025} = \frac{.025t \ln e}{.025}$$

$$\boxed{27.726 \text{ years} = t}$$

nominal rate = constant rate

effective rate - ending rate given compounding interest

$$R = \left(1+\frac{r}{n}\right)^{n} - 1$$

R = effective rate
r = nominal rate
n = number of compounding per year

Present value → Future value

$$P = A\left(1+\frac{r}{n}\right)^{-nt}$$

P = present value
A = future value
t = years
n = number compounds / year

Amortization - paying of principal and interest by a sequence of equal periodic payments

$$R = \frac{P}{\left[\frac{1-(1+i)^{-n}}{i}\right]}$$

P = loan amount
i = interest rate
n = # of payments
R = payments ea. period

Total Interest paid on the loan (I)

$$I = nR - P$$