5.4 Logarithmic Functions

Find the inverse of $f(x) = a^x$ analytically

$y = a^x \Rightarrow x = a^y$ switch x and y

$\log x = \log a^y$ log both sides

$\frac{\log x}{\log a} = \frac{y \log a}{\log a}$

$\log_x a = y$

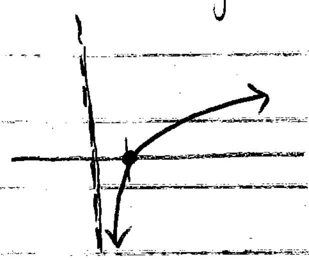
$\boxed{\log_x a = f^{-1}(x)}$

* Exponential and logarithmic functions are inverses

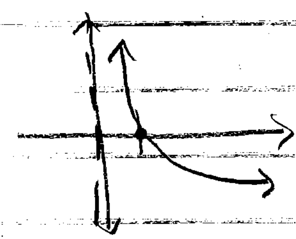
Graph of Logarithms $f(x) = \log_a x$

$a>0$

$\log x$
$\ln x$



$0<a<1$



Domain: $(0, \infty)$
Range: $\mathbb{R}$
Asymptote: $x=0$
X-intercept: $(1,0)$

Translations of $f(x) = \log_a x$

$f(x) = \log_a(x-b) + c$

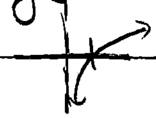
$-\log_a x \Rightarrow$ reflection over x-axis
$\log_a(-x) \Rightarrow$ reflection over y-axis

Graph reflections first

$b \Rightarrow$ left/right
$c \Rightarrow$ up/down

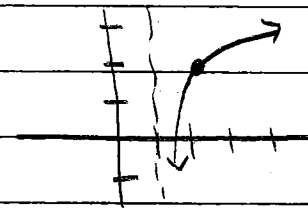
Example 1 List the transformations and graph

(A) $f(x) = \log_3(x-1)+2$

Parent 

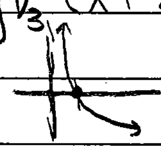
(−1) Right 1

(+2) up 2



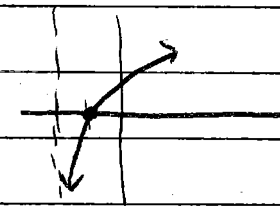
Domain: $x > 1$

(B) $f(x) = -\log_{1/3}(x+2)$

Parent 

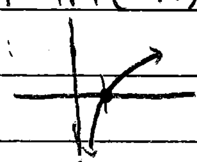
(−) Refl. x-axis

(+2) left 2



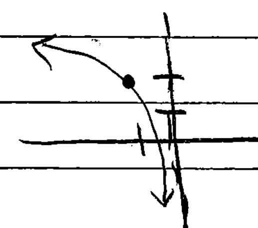
Domain: $x > -2$

(C) $f(x) = 2 + \ln(-x)$

Parent: 

(−) Refl. y-axis

(2) up 2



Domain: $x < 0$

Determining Domain of logarithms $f(x) = \log_a x$

Domain $x > 0$

Reference Example 1

(A) $\log_3(x-1)+2$

$\hookrightarrow x-1>0$

$x>0$

(B) $-\log_{1/3}((x+2))$

$\hookrightarrow x+2>0$

$x>-2$

(C) $2+\ln(-x)$

$\hookrightarrow -x>0$

$x<0$

$\boxed{5.4}$ Cont'd

Example 2: Determine the domain of $f(x)$

(A) $f(x) = \log(2x) - 1$ D: $2x > 0$

$x > 0$

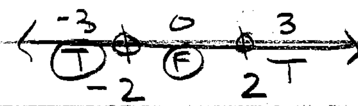
$(0, \infty)$

(B) $f(x) = \log(x^2 - 4)$ D: $x^2 - 4 > 0$

$x^2 > 4$

$x > \pm 2$

Test intervals:



$\boxed{\text{Domain: } (-\infty, -2)(2, \infty)}$

(-3) $(-3)^2 > 4$ (0) $0^2 > 4$

$9 > 4$ $0 > 4$

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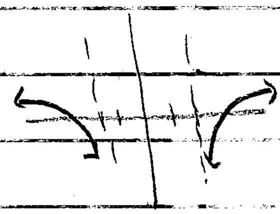
Graph:



(3) $3^2 > 4$

$9 > 4$

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(C) $f(x) = \ln(x^2 + 7)$ D: $\cancel{x}$ $x^2 + 7 > 0$

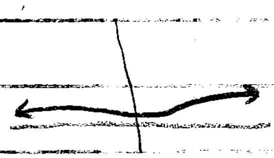
$x^2 > -2$

$\sqrt{x^2} > \sqrt{-2}$

undefined

$\boxed{\text{Domain: } \mathbb{R}}$

Graph



(D) $f(x) = \log|2x-1|$ D: $|2x-1| > 0$

$2x-1>0$ $\quad$ $-(2x-1)>0$

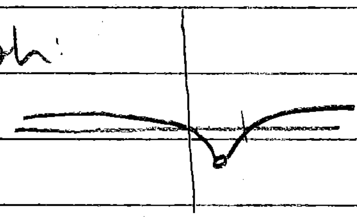
$2x>1$ $\quad$ $-2x+1>0$

$x>\frac{1}{2}$ $\quad$ $-2x>-1$

$x<\frac{1}{2}$

$\boxed{\text{Domain: } x \neq \frac{1}{2}}$

Graph:



Tblset: independent ASK

Table: $X = .5$ $Y =$ Error

(61) Suppose $f(x) = \log_a x$ and $f(3) = 2$. Find $f(\frac{1}{9})$

$f(x) = \log_a x$

$f(3) = \log_a 3$

$2 = \log_a 3$

Solve for a

$\sqrt{a^2} = \sqrt{3}$

$a = \sqrt{3}$

substitute a to find equation

$f(x) = \log_{\sqrt{3}} x$

$f(\frac{1}{9}) = \log_{\sqrt{3}} (\frac{1}{9})$

$y = \log_{\sqrt{3}} \frac{1}{9}$

Convert to exponential

$(\sqrt{3})^y = \frac{1}{9}$

$3^{\frac{1}{2}y} = 3^{-2}$

$\frac{\frac{1}{2}y}{\frac{1}{2}} = \frac{-2}{\frac{1}{2}}$

$y = -4 \Rightarrow \boxed{f(\frac{1}{9}) = -4}$