pg. 415 1-4, 13-27 odd, 29-57 EOO, 95, 105-109 odd, 113, 117, 133, (144)

5.4 Exponential and Logarithmic Equations

One-to-One property - If $a^x = a^y$, then $x = y$

converting between exponential / logarithmic

$$\log_B A = P \qquad B^P = A$$

Recall:

$$\log(ab) = \log a + \log b$$
$$\log\left(\frac{a}{b}\right) = \log a - \log b$$
$$\log a^n = n \log a$$

$$\log_b x = \log_b y \text{ iff } x = y$$
$$a^x = a^y \text{ iff } x = y$$

$\ln e = 1$	$\log 10 = 1$	$\log_a a = 1$
$\ln 1 = 0$	$\log 1 = 0$	$\log_a 1 = 0$
$\ln e^x = x$	$\log 10^x = x$	$\log_a a^x = x$

Example 1 Solve for x

(A) $\frac{1}{4}^x = 64$

$4^{-x} = 4^3$

$-x = 3$

$\boxed{x = -3}$

(B) $e^x = 4$

$\ln e^x = \ln 4$

$x \ln e = \ln 4$

$\boxed{x = \ln 4}$

(C) $\log x = -2$

$10^{-2} = x$

$\frac{1}{10^2} = x$

$\boxed{\frac{1}{100} = x}$

(D) $\log_5 x = \frac{1}{2}$

$5^{\frac{1}{2}} = x$

$\boxed{\sqrt{5} = x}$

Graphically - solutions are intersections
Intersections of $f(x)$ and $g(x)$ are located @ $f(x) = g(x)$

Example 2 Find the intersection algebraically and graphically

(A) $f(x) = 27^x$ $g(x) = 9$

$f(x) = g(x)$

$27^x = 9$

$3^{3x} = 3^2$

$3x = 2$

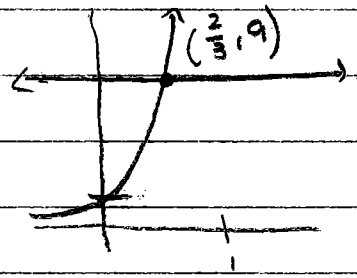
$\boxed{x = \frac{2}{3}}$

$f(\frac{2}{3}) = 27^{\frac{2}{3}}$

$= (\sqrt[3]{27})^2$

$= 9$

$(\frac{2}{3}, 9)$



Example 3 Solve Round to 3 decimals

(A) $e^{2x} = e^{x^2-8}$

$2x = x^2 - 8$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$\boxed{x = 4, -2}$

(B) $\frac{2(5^x)}{2} = \frac{32}{2}$

$5^x = 16$

$\frac{x \ln 5}{\ln 5} = \frac{\ln 16}{\ln 5}$

$\boxed{x = 1.723}$

* Do not round until the end *

5.4 cont'd

(C) $4^{-3t} = .10$

$\frac{-3t(\ln 4)}{\ln 4} = \frac{\ln .10}{\ln 4}$

$\frac{-3t}{-3} = \left(\frac{\ln .10}{\ln 4}\right) \div -3$

$t = .554$

(D) $8^{-2-x} = 431$

$(-2-x)\ln 8 = \ln 431$

$\frac{-2-x}{+2} = \frac{\ln 431}{\ln 8} + 2$

$-1(-x) = -\left(\frac{\ln 431}{\ln 8} + 2\right)$

$x = -4.917$

(E) $\frac{8(3^{6-x})}{8} = \frac{40}{8}$

$3^{6-x} = 5$

$\frac{(6-x)\ln 3}{\ln 3} = \frac{\ln 5}{\ln 3}$

$\frac{6-x}{-6} = \frac{\ln 5}{\ln 3} - 6$

$-1(-x) = -\left(\frac{\ln 5}{\ln 3} - 6\right)$

$x = 4.535$

(F) $8(4^{6-2x}) + 13 = 41$

$8(4^{6-2x}) = 28$

$4^{6-2x} = \frac{7}{2}$

$(6-2x)\ln 4 = \ln \frac{7}{2}$

$\frac{6-2x}{-6} = \left(\frac{\ln(3.5)}{\ln 4} - 6\right) \div -2$

$-x = -\left[\left(\frac{\ln 3.5}{\ln 4} - 6\right) \div 2\right]$

$x = 2.548$

⑥ $6\ln 3x = 15$

$\ln 3x = \frac{5}{2}$

$\frac{e^{2.5}}{3} = \frac{3x}{3}$

$4.061 = x$

Ⓗ $\log_3 x + \log_3(x-8) = 2$

$\log_3 x(x-8) = 2$

$x(x-8) = 3^2$

$x^2 - 8x - 9 = 0$

$(x-9)(x+1) = 0$

$x = 9 \quad x = -1$

$x = 9$

Extraneous Solution - solution that appears to be true but does not satisfy the original function.

Graphing Solutions are zeros (x-intercepts)

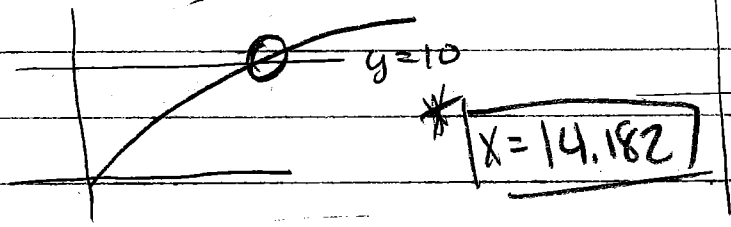
Example 4 Solve using a graphing calc

$10 = 4\ln(x-2)$

Intersection

$Y_1 = 10$

$Y_2 = 4\ln(x-2)$

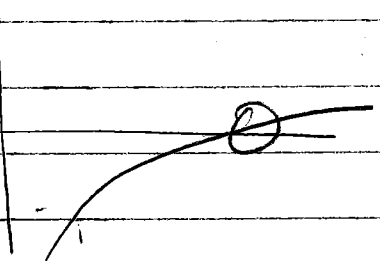


2nd | calc | intersect

(14.182, 10)

x-intercepts (zeros)

$Y_1 = 4\ln(x-2) - 10 = 0$



2nd | calc | zero

(14.182, 0)