PG. 405 7-19 EOO, 33-43 odd, 45-83 EOO

5.3 Properties of Logarithms

Change of base formula

$$\log_a x = \frac{\log x}{\log a} \quad \text{or} \quad \frac{\ln x}{\ln a}$$

Example 1 Rewrite the logarithm as a ratio of common log and natural logs. Evaluate to 3 decimals

(A) $\log_6 42$ $\qquad \frac{\log 42}{\log 6} =$ $\qquad \frac{\ln 42}{\ln 6} =$

Properties of logarithms

Product Property: $\log(ab) = \log a + \log b$
$\ln(ab) = \ln a + \ln b$

Quotient Property: $\log\left(\frac{a}{b}\right) = \log a - \log b$
$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

Power Property: $\log a^n = n \log a$
$\ln a^n = n \ln a$

Recall Properties of Logs:

$\log_a 1 = 0$

$\log_a a = 1$

$\log_a a^x = x$

If $\log_a x = \log_a y$, then $x = y$

Example 2 Rewrite each logarithm in terms of $\log 3$ and $\log 5$

Ⓐ $\log 15 = \log(3 \cdot 5)$
$= \log 3 + \log 5$

Ⓑ $\log_9 25 = \dfrac{\log 25}{\log 9} = \dfrac{\log 5^2}{\log 3^2} = \dfrac{2\log 5}{2\log 3}$
$= \boxed{\dfrac{\log 5}{\log 3}}$

Example 3 Rewrite and simplify

Ⓐ $\log_2(4^2 \cdot 3^4) = \dfrac{\log 4^2 + \log 3^4}{\log 2}$
$= \dfrac{2\log 2^2 + 4\log 3}{\log 2} = \dfrac{4\log 2}{\log 2} + \dfrac{4\log 3^4}{\log 2}$
$= 4 + 4\dfrac{\log 3}{\log 2} = \boxed{4(1 + \log_2 3)}$

Ⓑ $\log \frac{9}{300} = \log 9 - \log 300$
$= \log 3^2 - \log(3 \cdot 100)$
$= 2\log 3 - (\log 3 + \log 100)$
$= 2\log 3 - \log 3 - \log 10^2$
$= \log_3 - 2\log 10$
$= \boxed{\log_3 - 2}$

5.3 cont'd

Example 4 Find the exact value without a calc

(A) $\log_5 \frac{1}{125}$

(1) using laws of logs

$$\frac{\log \frac{1}{125}}{\log 5} = \frac{\log 5^{-3}}{\log 5} = \frac{-3\log 5}{\log 5} = \boxed{-3}$$

(2) set equal to x and change to exponential

$\log_5 \frac{1}{125} = x$

$5^x = \frac{1}{125}$

$5^x = 5^{-3}$

$x = -3$

$\boxed{-3}$

* Answer should not have the variable included

(B) $\log_3 72 - \log_3 8 = \log_3\left(\frac{72}{8}\right)$
$= \log_3 9$

(1) $\log_3 3^2 = \boxed{2}$

(2) $\log_3 9 = x$

$3^x = 9$ $\boxed{2}$

$3^x = 3^2$

(3) $\frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2\cancel{\log 3}}{\cancel{\log 3}} = \boxed{2}$

(C) $\log_3 81^{-3}$

(1) laws of logs

$$\frac{\log 81^{-3}}{\log 3} = \frac{\log (3^4)^{(-3)}}{\log 3} = \frac{-12\cancel{\log 3}}{\cancel{\log 3}} = \boxed{-12}$$

(2) Exponential

$3^x = 81^{-3}$

$3^x = (3^4)^{-3}$

$3^x = 3^{-12}$

$x = -12$

$\boxed{-12}$

Example 5 Use properties of logs to expand

(A) $\log_7 \frac{1}{x^4} = \log_7 1 - \log_7 x^4$

$= 0 - 4\log_7 x$

$= \boxed{-4\log_7 x}$

(B) $\ln\sqrt{x^2(x+2)} = \ln\left(x^2(x+2)\right)^{\frac{1}{2}}$

$= \ln\left(x(x+2)^{\frac{1}{2}}\right)$

$= \boxed{\ln x + \frac{1}{2}\ln(x+2)}$

Example 6 Condense to a single quantity

(A) $2\ln 8 + 5\ln(z-4) = \ln 8^2 + \ln(z-4)^5$

$= \boxed{\ln\left(64(x-4)^5\right)}$