$\boxed{5.3}$ Logarithms and Their Properties

Identify the base, answer, and power

	$2^3 = 8$	$(x+1)^4 = 256$	$3^{x-4} = 27$
Base:	2	$x+1$	3
Answer:	8	256	27
Power:	3	4	$x-4$

Exponential $B^P = A$ Logarithm $\log_B A = P$

Convert the above examples to logarithm

$2^3 = 8$ $(x+1)^4 = 256$ $3^{x-4} = 27$

$\log_2 8 = 3$ $\log_{x+1} 256 = 4$ $\log_3 27 = x-4$

Common Logarithm $\log x = \log_{10} x$

$10^3 = 1000 \Rightarrow \log 1000 = 3$

$\log 100 = 2 \Rightarrow 10^2 = 100$

Natural logarithm $\ln x = \log_e x$

$e^2 = 7.389$ $\ln 7.389 = 2$

$\ln 5 = 1.609$ $e^{1.609} = 5$

Properties of Logarithms

Product: $\log_a xy = \log_a x + \log_a y$

Quotient: $\log_a \frac{x}{y} = \log_a x - \log_a y$

Power: $\log_a x^y = y \log_a x$

Exponential

$a^x a^y = a^{x+y}$

$\frac{a^x}{a^y} = a^{x-y}$

$(a^x)^y = a^{xy}$

Example 1 Use properties of logarithms to rewrite each expression.

(71) (A) $\log_2 \frac{6x}{y} = \log_2 6x - \log_2 y$

$= \log_2 6 + \log_2 x - \log_2 y$

(77) (B) $\log_k \frac{pq^2}{m} = \log_k pq^2 - \log_k m$

$= \log_k p + \log_k q^2 - \log_k m$

$= \log_k p + 2\log_k q - \log_k m$

Example 2 Use properties of logarithms to rewrite as a single log with coefficient 1.

(82) (A) $(\log_b k - \log_b m) - \log_b a = \log_b \frac{k}{m} - \log_b a$

$= \log_b \frac{\frac{k}{m}}{\frac{a}{1}}$ $\qquad \frac{k}{m} \cdot \frac{1}{a}$

$= \log_b \frac{k}{ma}$

(85) (B) $2\log_a(z-1) + \log_a(3z+2) = \log_a(z-1)^2 + \log_a(3z+2)$

$= \log_a (z-1)^2(3z+2)$

(91) (C) $\ln(a+b) + \ln a - \frac{1}{2}\ln 4 = \ln(a+b) + \ln a - \ln 4^{\frac{1}{2}}$

$= \ln(a+b) + \ln a - \ln\sqrt{4}$

$= \ln(a+b)a - \ln 2$

$= \ln \frac{a(a+b)}{2}$

5.3 cont'd

Change of Base Rule $\log_a x = \frac{\log x}{\log a}$ or $\frac{\ln x}{\ln a}$

Common Base Rule If $a^x = a^y$ then $x = y$

If $\log_a x = \log_a y$, then $x = y$

Example 2 Solve

19 A $\log_5 125 = x$

$5^x = 125$ change to exponential

$5^x = 5^3$ find common bases

$\boxed{x = 3}$ Common base rule

$\log_5 125 = x$

$\frac{\log 125}{\log 5} = x$ Change of base Rule

$3 = x$

25 B $\log_x 16 = \frac{4}{3}$

$x^{\frac{4}{3}} = 16$

$x^{\frac{4}{3} \cdot \frac{3}{4}} = 16^{\frac{3}{4}}$

$x = (\sqrt[4]{16})^3$

$x = 2^3$

$\boxed{x = 8}$

28 C $\log_3 (x-1) = 2$

$3^2 = x - 1$

$9 = x - 1$

$\boxed{10 = x}$

Example 3 Simplify

(31) (A) $3^{\log_3 7} = x$

$\log_3 x = \log_3 7$

$x = 7$

$\boxed{7}$

we added the variable x. Don't include in the answer

(B) $4^{\log_4 9} = x$

$\log_4 x = \log_4 9$

$x = 9$

$\boxed{9}$

(D) $a^{\log_a k} = \boxed{k}$

Example 4 Evaluate

(35) (A) $\log 10^{1.5} = x$

$10^x = 10^{1.5}$

$x = 1.5$

$\boxed{1.5}$

(B) $\ln e^{\sqrt{6}} = x$

$\log_e e^{\sqrt{6}} = x$

$e^x = e^{\sqrt{6}}$

$\boxed{\sqrt{6}}$