5.2 Logarithmic Functions and Their Graphs

Logarithmic Function - inverse of exponential

Logarithmic	⇒	Exponential
$\log_B A = P$		$B^P = A$

Common log - base 10

$\log x = y \Rightarrow 10^y = x$

Natural log - base e

$\ln x = y \Rightarrow e^y = x$

Example 1 Convert between logarithmic and exponential

(A) $\log_7 343 = 3$

$7^3 = 343$

$\log_7 343 = 3$

$\log_B A = P$

$7^3 = 343$

(B) $\log_{1000} = -3$

$10^{-3} = 1000$

$\log_{1000} = -3$

$\log_B A = P$

$10^{-3} = 1000$

(C) $\log_{16} 8 = \frac{3}{4}$

$16^{\frac{3}{4}} = 8$

$* \left(\sqrt[4]{16}\right)^3 = 8$

(D) $13^2 = 169$

$\log_{13} 169 = 2$

$13^2 = 169$

$B^p = A$

$\log_{13} 169 = 2$

(E) $4^{-3} = \frac{1}{64}$

$\log_4 \frac{1}{64} = -3$

(F) $e^4 = x$

$\log_e x = 4$

$\ln x = 4$

(G) $\ln 10 = 2.302$

$e^{2.302} = 10$

Example 2 Evaluate without a calculator

(A) $f(x) = \log_{25} x \qquad x = 5$

Rewrite as exponential and substitute $x=5$

$y = \log_{25} 5 \Rightarrow 25^y = 5$

Use one-to-one if possible $\Rightarrow$ $25^y = 5^1$

$(5^2)^y = 5^1$

$2y = 1$

$y = \boxed{\frac{1}{2}}$

(B) $f(x) = \log x \qquad x = 100$

$y = \log 100 \Rightarrow 10^y = 100$

$10^y = 10^2$

$y = \boxed{2}$

5.2 cont'd

Properties of Logs / Natural Logs

Logarithms

$\log_a 1 = 0 \Rightarrow a^0 = 1$
$\log_a a = 1 \Rightarrow a^1 = a$
$\log_a a^x = x \Rightarrow a^x = a^x$

Natural logs

$\ln 1 = 0 \Rightarrow e^0 = 1$
$\ln e = 1 \Rightarrow e^1 = e$
$\ln e^x = x \Rightarrow e^x = x$

One-to-One Properties

If $\log_a x = \log_a y$, then $x = y$
If $\ln x = \ln y$, then $x = y$

Example 3 Use properties of logs to simplify

(A) $\log_{3.2} 1$

Add an x in order to change to exponential

$\log_{3.2} 1 = x$
$3.2^x = 1$
$3.2^x = 3.2^0$
$x = 0$

Do not include x in the answer: $\boxed{0}$

(B) $9^{\log_9 15}$

$9^{\log_9 15} = x$

$\log_9 x = \log_9 15$

$x = 15$

$\boxed{15}$

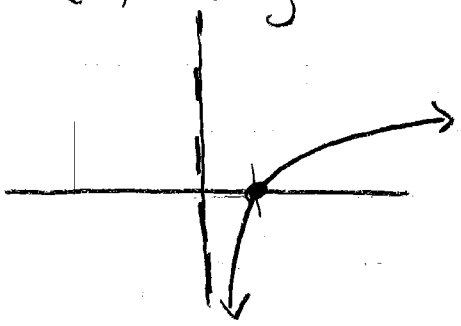
(C) $\log_2(x-3) = \log_2 9$

$x - 3 = 9$

$x = 12$

Graphs of logarithms

$f(x) = \log x$



Critical points:

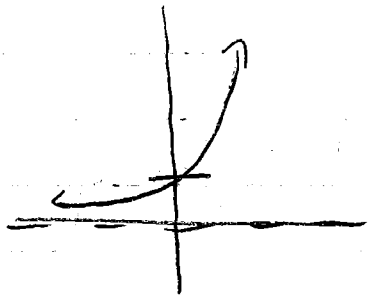
(1,0)

vertical asymptote: $x=0$

Domain: $(0, \infty)$

Range: $\mathbb{R}$

* inverse of exponential $f(x) = a^x$



(0,1)

horizontal asymptote $y=0$

Domain: $\mathbb{R}$

Range: $(0, \infty)$

Domain - Find the domain

(A) $\log(2-x)$

$2 - x > 0$

$2 > x$

$\boxed{x < 2}$

(B) $3\log(2x+1) - 3$

$2x + 1 > 0$

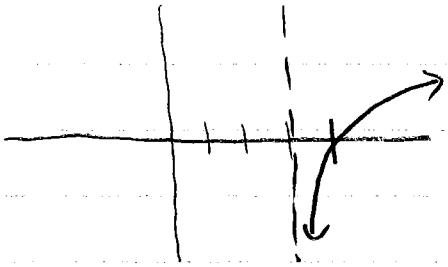
$2x > -1$

$\boxed{x > -\frac{1}{2}}$

5.2 cont'd

Example 4 Find the domain, x-intercept, asymptote and sketch the graph

(A) $f(x) = \log_4(x-3)$



Domain: $x-3>0$

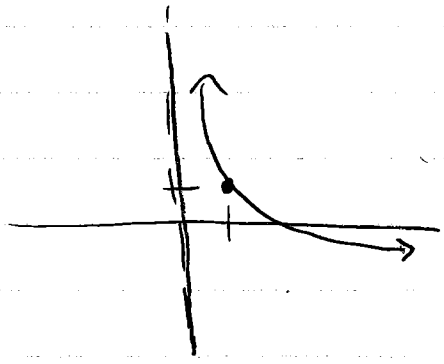
$x>3$

Transformation: Right 3

x-intercept: $(4,0)$

asymptote $x=3$

(B) $f(x) = -\ln x + 1$



Domain: $x>0$

Transformation

Reflection over x-axis

up 1

x-intercept: $(e,0)$

asymptote: $x=0$

x-intercept:

$0 = -\ln x + 1$

$-1 = -\ln x$

$1 = \ln x$

$e^1 = x$

$e = x$