5.1 Inverse Functions Part II

Calc

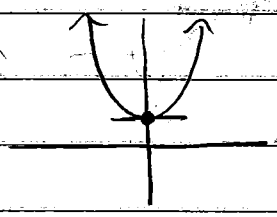
Open Determine if the function is one-to-one

Ⓐ x^3-1 yes Ⓑ x^2+2x-1 no Ⓒ x^4+6x no Ⓓ x^5-3x^2+2 yes

* Even degree functions will not be one-to-one

Restricting the Domain - needed to create a one-to-one function to find an inverse

① Graph $f(x) = x^2 + 1$



② Find the inverse of $f(x)$ analytically

$y = x^2 + 1$

$x = y^2 + 1$

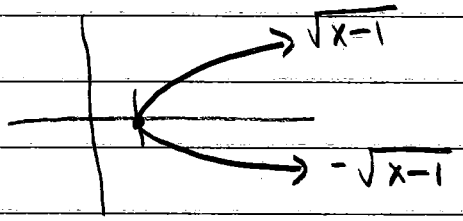
$x - 1 = y^2$

$\pm\sqrt{x-1} = y$

③ Graph $y = \sqrt{x-1}$

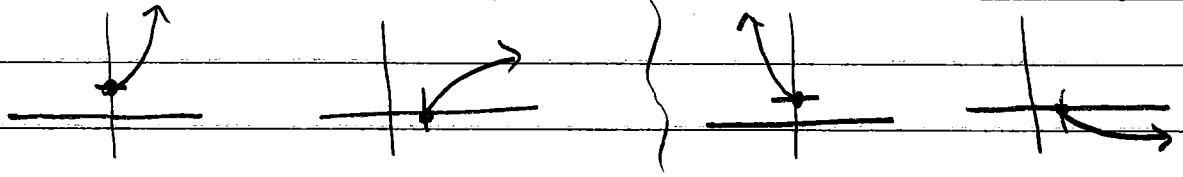
$y = -\sqrt{x-1}$

not a function



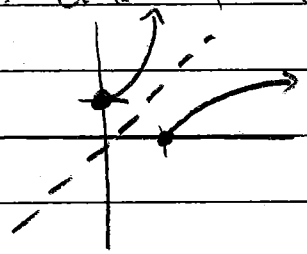
④ Restrict the domain of $f(x) = x^2 + 1$

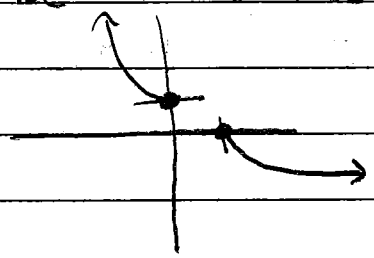
$x \geq 0$ $f^{-1}(x) = \sqrt{x-1}$ $x \leq 0$ $f^{-1}(x) = -\sqrt{x-1}$



Ex. 1 Use the restrictions to find $f^{-1}(x)$ $f(x) = 2|x-3|$ $x \geq 3$

$f(x)$ and $f^{-1}(x)$ will be reflections over $y = x$

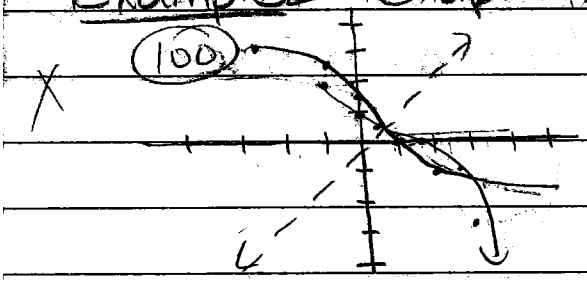




$f(x) = x^2 + 1$ $x \geq 0$ $f(x) = x^2 + 1$ $x \leq 0$

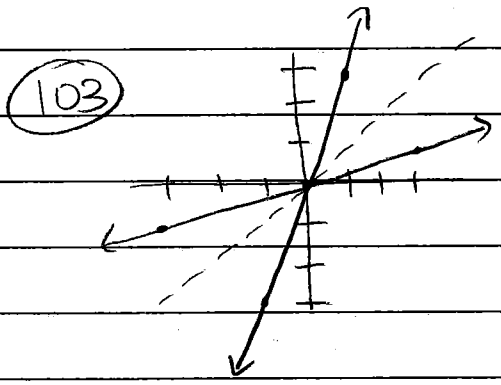
Example 2 Graph the inverse of the function

coordinates of inverses switch (x, y)



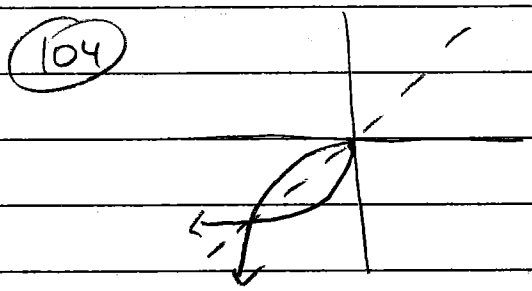
$f(x)$	x	.5	1	2	0	-1	-3
	y	.5	0	-1	1.5	2.5	3

$f^{-1}(x)$	x	.5	0	-1	1.5	2.5	3
	y	.5	1	2	0	-1	-3



$f(x)$	x	0	1	-1
	y	0	3	-3

$f^{-1}(x)$	x	0	3	-3
	y	0	1	-1

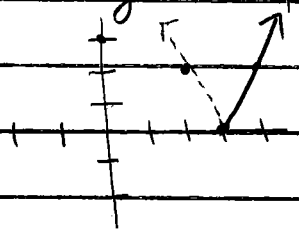


5.1 cont'd

Example 1 Use the restrictions on the domain to find a formula for $f^{-1}(x)$

$f(x) = 2|x-3| \quad x \geq 3$

right 3, vertical stretch of 2 (slope = 2)



$x \geq 3$ positive abs value

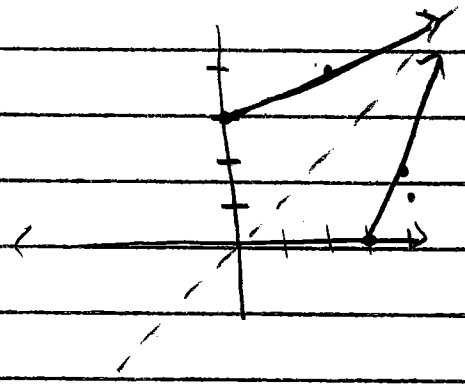
$f(x) = 2(x-3)$

$= 2x - 6$

$x = 2y - 6$

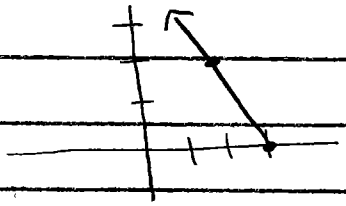
$x + 6 = 2y$

$\frac{x+6}{2} = y$



$\frac{1}{2}x + 3 = f^{-1}(x) \quad \{y \geq 3\}$

$f(x) = 2|x-3| \quad x \leq 3$



$x \leq 3$ negative slope

$f(x) = 2[-(x-3)]$

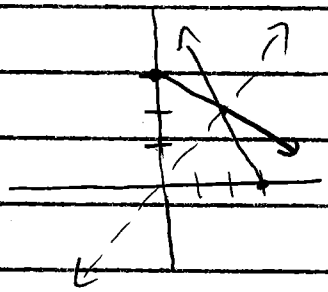
$= 2(-x+3)$

$= -2x + 6$

$x = -2y + 6$

$\frac{x-6}{-2} = \frac{-2y}{-2}$

$-\frac{1}{2}x + 3 = y$



$-\frac{1}{2}x + 3 = f^{-1}(x) \quad y \leq 3$