Syllabus Open/Close
Logs
Who Are You?

5.1 Inverse Functions Part I

Open Example 8 $f(x) = 2x - 3$

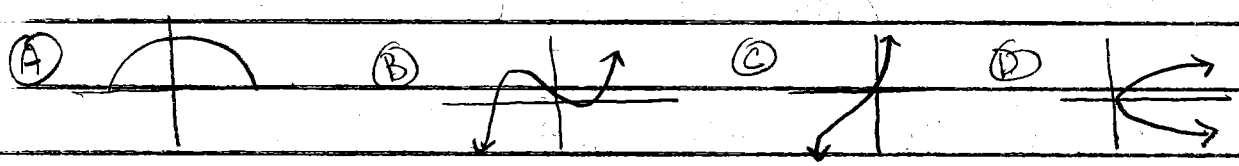
Decode Answer: Miss Ojeda

23 15 35 35 27 17 3 5 -1

Vertical line test Determines if a graph is a function
Horizontal line test Determines if the inverse is a function

One-to-One function Each input corresponds to only one output and each output corresponds to one input

Example 1 Function? Inverse Function? One-to-One?



Determining one-to-one algebraically

① Show if $f(a) = f(b)$, Then $a = b$

(or)

② Find two different values that produce the same output

(or)

③ Does the graph pass the vertical and horizontal line test

Example 2 Decide whether the function is one-to-one

⑤ $f(x) = \sqrt{36 - x^2}$

① $f(a) = f(b)$

$\sqrt{36 - a^2} = \sqrt{36 - b^2}$

$36 - a^2 = 36 - b^2$

$-a^2 = -b^2$

$a^2 = b^2$

$\pm a = \pm b$

not one to one

② $f(1) = \sqrt{36 - 1^2}$

$= \sqrt{35}$

$f(-1) = \sqrt{36 - (-1)^2}$

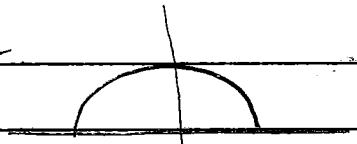
$= \sqrt{36 - 1}$

$= \sqrt{35}$

$f(1) = f(-1)$

not one-to-one

③ Graph



not one to one

* Analytically/Algebraically ① and ② Graphically ③

Inverse Functions $f(x)$ and $f^{-1}(x)$ are inverses if:

① The domain of $f(x)$ is the range of $f^{-1}(x)$ and the range of $f(x)$ is the domain of $f^{-1}(x)$

② Point (a,b) lies on the graph of $f(x)$ and the point (b,a) lies on the graph of $f^{-1}(x)$

③ $(f \circ g)(x) = x$ And $(g \circ f)(x)$

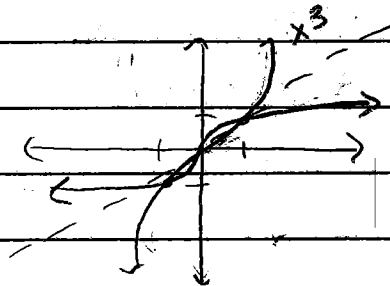
④ $f(x)$ must be one-to-one for $f^{-1}(x)$ to exist

⑤ $f^{-1}(x)$ is a reflection of $f(x)$ over the line $y = x$

Calculator

⑥ $f(x) = x^3$

$f^{-1}(x) = \sqrt[3]{x}$



5.1 Part I cont'd

Finding inverse functions - switch x and y in f(x), Then solve for y.

* f(x) must be one-to-one for $f^{-1}(x)$ to exist

Example 3 Find $f^{-1}(x)$

(74) $f(x) = 4 - 3x^3$

$y = 4 - 3x^3$ Switch x & y

$x = 4 - 3y^3$ solve for y

$\frac{x-4}{-3} = \frac{-3y^3}{-3}$

$\sqrt[3]{\frac{x-4}{-3}} = \sqrt[3]{y^3}$

$$f^{-1}(x) = \sqrt[3]{\frac{x-4}{-3}}$$

(67) $f(x) = \frac{4x}{x+1}$

$(y+1)\,x = \frac{4y}{y+1}(y+1)$ Switch x & y

$xy + x = 4y$ get y's together on one side

$x = 4y - xy$ factor y

$x = y(4-x)$ Solve for y

$\frac{x}{4-x} = y$

$$f^{-1}(x) = \frac{x}{4-x}$$

Example 4 Show analytically that f and g are inverses must show $(f \circ g)(x) = x$ AND $(g \circ f)(x) = x$

(37) $f(x) = 3x - 7$

$g(x) = \frac{x+7}{3}$

$(f \circ g)(x) = 3\left(\frac{x+7}{3}\right) - 7$

$= x + 7 - 7$

$= x$

$(g \circ f)(x) = \frac{3x - 7 + 7}{3}$

$= \frac{3x}{3}$

$= x$

(40) $f(x) = x^3 - 7$

$g(x) = \sqrt[3]{x+7}$

$(f \circ g)(x) = \left(\sqrt[3]{x+7}\right)^3 - 7$

$= x + 7 - 7$

$= x$

$(g \circ f)(x) = \sqrt[3]{x^3 - 7 + 7}$

$= \sqrt[3]{x^3}$

$= x$