

## 13.1 Comparing Two Means

### Paper Airplane Experiment (Tape Measure)

#### Lipitor vs. Pravachol

$n = 4000$  people with heart disease

Randomly assigned to two groups: L or P

① Bad cholesterol levels are tested

Is the difference  
statistically significant?

Comparing two means:  $P(\bar{x}) = 95 \text{ mg/dl}$

$L(\bar{x}) = 62 \text{ mg/dl}$

② Patients who died, heart attack, or other  
serious condition within two years

Is the difference  
purely by chance?

Comparing two proportions:  $P(\hat{p}) = .263$

$L(\hat{p}) = .224$

### Two-sample problems - Comparing two populations or two treatments

- Goal of inference is to compare the responses to two treatments or to compare the characteristics of two populations

Example B1 (pg 781)

Examples of

two-sample studies

- Separate sample from each treatment or each population.

- Responses of each group are independent of those in the other group

### Matched Pairs vs. Two-Sample

Matched pairs matches samples; same size, traits, etc

Two samples do not require similarity, even in size

HW 13.2, 13.4 first (?)

### Conditions for Comparing Two Means

SRS/Independence - Independent, random samples from the populations of interest or data from a randomized two-treatment experiment

- Normality
- ①  $n_1, n_2 \geq 30$  CLT says distribution is  $\sim N$
  - ② Graphs of sample data is  $\sim N$
  - ③ Stated as  $\sim N$

Slide-by-slide w/ steps

### Steps for two sample t-test for difference in means

- ① State the hypothesis:  $H_0: \mu_1 - \mu_2 = 0$   
 $H_a: \mu_1 - \mu_2 \neq 0$

- ② Procedure: Two sample t-test for the difference in mean (context) for all (context)

- ③ Check conditions: S, I, N

- ④ Computation:  $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$   $df =$   
 $\frac{(z = \text{same})}{\text{w/o df}}$   $p\text{-value} =$

- ⑤ Conclusion: Since the p-value is  $\leq \alpha$  I reject/fail to reject  $H_0$  that (context). There is/is not evidence to conclude (context).

### Degrees of Freedom Two-Sample t stat

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1} \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1} \left(\frac{s_2^2}{n_2}\right)^2}$$

Calculator: **STAT** **TESTS**  
2-SampTTest

w/o technology: use smaller of  $n_1 - 1$  and  $n_2 - 1$

### 13.1 cont'd

Calculator Technology toolbox pg. 795  
Significance test  
Confidence Intervals

Pooling - procedure averaging (pooling) two sample variances to estimate the common population variance. Populations are assumed to have equal variances in a pooled test. Since variances are rarely equal in a two-sample test, DO NOT Pool

Steps for a two-sample  $t$  CI:

① Procedure: Two-sample  $t$  CI for  $\mu_1 - \mu_2$  where  $\mu_1 - \mu_2 =$  the difference in mean for all \_\_\_\_\_ ( $\mu_1$ ) and all \_\_\_\_\_ ( $\mu_2$ )

② Check conditions. (same as two-sample  $t$ )

③ Computation:  $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

$$t^* = \text{invT}(\text{area}, \text{df})$$

④ Conclusion: Based on these samples, I am \_\_\_\_\_ % confident that the true difference in mean \_\_\_\_\_ for all \_\_\_\_\_ and all \_\_\_\_\_ is between \_\_\_\_\_ and \_\_\_\_\_.

### Example 1 Tangrams

Andy's Puzzle:  $\bar{X} = 157.765$   $s = 116.762$   $n = 17$

Volcano Puzzle:  $\bar{X} = 93.231$   $s = 66.708$   $n = 27$

① Do the data provide convincing evidence at the  $\alpha = .05$  level, that the average number of clicks used to solve the tangram is significantly higher for Andy's Puzzle than for the Volcano puzzle.

①  $H_0: \mu_A - \mu_V = 0$      $H_a: \mu_A - \mu_V \neq 0$

② Two-sample t-test for the difference in the mean number of clicks needed to complete Andy's Puzzle ( $\mu_A$ ) and the Erupting Volcano ( $\mu_V$ ) for all students who complete both puzzles.

③ We will consider the samples as independent and random. Both sample sizes (17 and 27) are  $> 15$ , so the CLT says the sampling is  $\sim N$ .

④  $t = (157.765 - 93.231) - 0$

$$\frac{116.762^2}{17} + \frac{66.708^2}{27} \approx 2.076$$

$$p\text{-value} = P(t > 2.076) = .025$$

$$df = 22.663 \text{ (calc)}$$

⑤ Since the p-value  $.025 < \alpha = .05$ , we reject  $H_0$ . There is evidence to conclude the average number of clicks used to solve the tangram is significantly higher for Andy's puzzle than the volcano.

### 13.11 cont'd

Slide 13.1

#### Example 2 Fast food

McDonald's average service time was 188.83 seconds with a standard deviation of 17.38 sec for a random sample of 362 drive-thru visits. Burger King on a separate random sample of 318 visits resulted in a mean of 201.33 seconds and a standard deviation of 18.85 sec.

Construct and interpret a 99% CI for the difference in mean services times at McD and BK.

Procedure

① Two-sample  $t$  confidence interval for  $\mu_m - \mu_b$  where  $\mu_m - \mu_b =$  the difference in mean service times for all McD ( $\mu_m$ ) drive-thrus and all BK ( $\mu_b$ ) drive-thrus

Conditions

② It is stated these are independent, random samples from all McD's and BK drive thrus. Since  $362 > 30$  and  $318 > 30$ , the CLT says the sampling distribution will be  $\sim N$ .

Computation

$$\textcircled{3} (\bar{X}_m - \bar{X}_b) \pm t^* \sqrt{\frac{S_m^2}{n_m} + \frac{S_b^2}{n_b}}$$
$$(188.83 - 201.33) \pm 2.583 \sqrt{\frac{17.38^2}{362} + \frac{18.85^2}{318}}$$

$$(-16.11, -8.891)$$
$$df = 649.284$$

StatTest  
2samp T test  
F = 649.284  
InvT (.995, 649.284)

(4) Based on these samples, I am 99% confident that the true difference in mean service time for all McD's and all BK drive-thrus (McD-BK) is between -16.11 and -8.891 seconds