Pg. 745 1-4, 8, 9abc
10, 12, 17, 19a, 21

$\boxed{12.1}$ Tests About a Population Mean (Day 1)

<u>One-sample t statistic</u> $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ $df = n-1$

t-table
Right tail

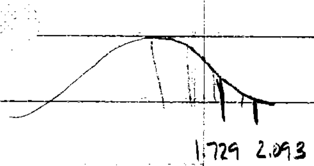
<u>Using the t-table Find each of the following critical values</u>

① $df = 11$, <u>probability of .05</u>, right of t, <u>find</u> t^*
$\boxed{t = 1.796}$

② $n = 18$, probability of .9, <u>left of t</u>, <u>find</u> t^*
$df = 17$ .10 to the left $\boxed{t = 1.333}$

12.1 slides

③ testing $H_0: \mu = 5$ $H_a: \mu > 5$, $n = 20$, $t = 1.81$
<u>find the p-value</u>



$df = 19$; 1.81 is between 1.729 and 2.093
$\boxed{.025 < \text{p-value} < .05}$

④ testing $H_0: \mu = 5$ $H_a: \mu \neq 5$, $n = 37$, $t = -3.17$
<u>find the p-value</u>
$df = 36$ 3.17 is between 3.030 and 3.385
$.001(2) < p < .0025(2)$
$\boxed{.002 < p < .005}$

<u>Calculator</u>

Calculator
left tail

① invT(.95, 11) ⇒ $t = 1.796$

② invT(.90, 17) ⇒ $t = 1.33$

③ tcdf(1.729, 1000, 19) ⇒ .05 $.025 < p < .05$
tcdf(2.093, 1000, 19) ⇒ .025
tcdf(1.81, 1000, 19) ⇒ $\boxed{.043}$

invT(area, df)
tcdf(lower, upper, df)

④ 2tcdf(3.17, 1000, 36) = 2(.0016) = $\boxed{.0032}$
(−1000, −3.17, 36)

HW) 12.4 $H_0: \mu = 64$ $H_a: \mu \neq 64$ $n = 25$ $t = 1.12$

(a) $df = 24$

(b) 1.12 is between 1.059 ($P = .15$) and 1.318 ($P = .10$)

(c) $H_a: \mu \neq 64 \Rightarrow 2(.10) < p < 2(.15)$

$$\boxed{.20 < p < .30}$$

Actual p-value $\Rightarrow$ 2tcdf(1.12, 1000, 24) = 2(.1369)

$$\boxed{p = .2738}$$

(d) no. $t = 1.12$ is not significant at $\alpha = .10$ or $\alpha = .05$

Steps in a One-sample t test for μ

1. State the hypotheses: $H_0: \mu = \mu_0$ $H_a: \mu \ (<, >, \neq) \ \mu_0$
2. State the procedure in context
   One-sample t-test for μ where μ = the true mean (context) of all (context) ____
3. State and Check conditions
   SRS - randomized experiment - stated in the question
   Independence - $N \geq 10n$
   Normal - Sampling distribution of $\bar{x}$ is $\sim N$
   stated in the question population is $\sim N$
   graph of the sample data is $\sim N$
   CLT if $n \geq 30$
4. Computation: Find the test statistic, p-value, & df
   $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$
   if $H_a <$ $P(t < t_0)$
   if $H_a >$ $P(t > t_0)$
   if $H_a \neq$ $2P(t > |t_0|)$
5. Conclusion: Because the p-value ___ is (< or >) α = ___, I (reject/fail to reject) the H_0. There (is/is not) convincing evidence that (context)

$\boxed{12.1}$ Cont'd (Day 1)

Example 12.2 Calculator T-test

L_1: Data from 12.2

$\boxed{STAT}\boxed{TESTS}\boxed{T\text{-}test}$ Data

$\mu_0 = 0$ List: L_1 Freq: 1 μ: $> \mu_0$

$t = 2.7$ $p = .012$ $\bar{x} = 1.02$ $S_x = 1.196$ $n = 10$

Proceed with caution. must assume normality and independence

(4) Calculation: $\bar{x}_{DIFF} = 1.02$ $S_{DIFF} = 1.196$

$$t = \frac{\bar{x} - \mu_0}{S/\sqrt{n}} = \frac{1.02 - 0}{1.196/\sqrt{10}} = 2.7$$

P-value for $t = 2.70$ is the area to the right of 2.70 with $df = 10 - 1 = 9$.

{ tcdf(2.70, 1000, 9) = .0122
{ Table C: observed t lies between 2.398 and 2.821
The p-value is between .01 and .02.

(5) Conclusion: Because the p-value .012 is $< \alpha = .05$, I reject the null hypothesis. There is enough evidence to conclude the cola has lost sweetness during storage.

* Example 12.3 Diversify or be sued

Calculator T-test: $\mu_0 = .95$ $\neq \mu_0$

$t = -2.137$ $p = .039$ $\bar{x} = -1.10$ $S_x = 5.99$ $n = 39$

may state context in hyp. or procedure

(1) Hypothesis: $H_0: \mu = .95$ $H_a: \mu \neq .95$ where μ is the true mean return for all months on the account

(2) One-sample t-test for μ, where μ is the true mean on the return for all possible months that the broker could manage this account.

(3) SRS is stated in the question. Independence is a matter of judgment and we must be willing to treat the 39 months as independent observations. Normality is satisfied since $n = 39 > 30$. (A graph of the data will also show normality)

(4) Calculations: $\bar{x} = -1.1$ $\quad S_x = 5.99$

$$t = \frac{-1.1 - .95}{5.99/\sqrt{39}} = \boxed{-2.14}$$

$2 \cdot \text{tcdf}(2.14, 1000, 38) = 2(.019)$

$p = \boxed{.039}$ $\quad \boxed{df = 38}$

(5) Conclusion: Because the p-value .039 is $< \alpha = .05$, we reject the H_0. There is convincing evidence that the investment for this client's account differs significantly from the S&P 500 for the same period.

Confidence Interval $\quad \bar{x} = -1.1\%$ $\quad S_x = 5.99\%$ $\quad$ 95% CI

text uses $t^* = 2.042$ Table C, df = 30

Table C: 95% df = 30 ⇒ 2.042

Calc: invT(.975, 38) = 2.024

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}} = -1.1 \pm 2.042\left(5.99/\sqrt{39}\right) = (-3.06, .86)$$

$$= -1.1 \pm 2.024\left(5.99/\sqrt{39}\right) = (-3.04, .841)$$

Because the S&P 500 return .95% falls outside this interval, we know μ differs significantly at the $\alpha = .05$ level.

12.1 Tests About a Population Mean (Day 2)

Day 2 notebook

Example 1 Average Body Temperature

In the early 1990's, researchers conducted a study to determine whether the accepted value for normal body temperature 98.6°, is accurate. They used an oral thermometer to measure the temperatures of an SRS of 130 healthy men and women aged 18 to 40.

Facts of the data set:

$\bar{x} = 98.25°$ $s = .73°$

62.3% of the readings were less than 98.6°

Do the data from this study provide evidence at the $\alpha = .01$ level that average normal body temperature of healthy 18-40 yo is not 98.6°F

1. $H_0: \mu = 98.6°F$ $H_a: \mu \neq 98.6°F$
2. One sample t test for μ where μ is the true average body temp of all healthy 18-40 year olds.
3. SRS is stated in the question. $N \geq 10(130) = 1300$. Because there are more than 1300 healthy 18-40 year olds independence is satisfied. $n = 130 > 30$, the CLT says the sampling distribution is $\sim N$.
4. $t = \dfrac{98.25 - 98.6}{.73/\sqrt{130}} = -5.467$

$2P(t < -5.467) \approx 0$ $df = 130 - 1 = 129$

⑤ Because the p-value is $\approx 0 < \alpha = .01$, we reject H_0
There is evidence that the average body temperature for all healthy 18-40 year olds is not 98.6°F

Example 2 Nitrogen in tires (Mathed Pairs)
Do the data give convincing evidence that filling tires with nitrogen instead of air decreases pressure loss?

① $H_0: \mu_D = 0$ $\quad H_a: \mu_D < 0$

② One sample t-test for μ_D where μ_D is the true mean difference (nitrogen - air) in tire pressure for all brands of tires.

③ We must assume this was a randomized experiment so the differences in tire pressure are random.
$N \geq 10n = 10(31) = 310$ Since there are > 310 pairs of tires, we can assume that the differences in tire pressure are independent. $n = 31 > 30$ so the CLT says the sampling distribution will be $\sim N$

④ $t = \dfrac{-1.252 - 0}{1.202/\sqrt{31}} = -5.797$ $\quad$ P-value $= P(t < -5.797) \approx 0$
$df = 31 - 1 = 30$

⑤ Because the P-value $\approx 0 < \alpha = .05$, we reject H_0.
There is evidence to conclude that filling tires with nitrogen instead of air decreases pressure loss.

Calculator: L_1: Air $\quad L_2$: Nitrogen $\quad L_3: L_2 - L_1$
T-Test: L_3; $\mu_0 = 0$; $\mu < \mu_0$
$t = -5.797$ $\quad p = 1.232$ $\quad \bar{x} = -1.252$ $\quad S_x = 1.202$ $\quad n = 31$

review pg. 655 using t-procedures

12.1 cont'd

Example 12.6 Power, Type I & Type II Errors
(Investment portfolio example 12.3)

Type I error: reject $H_0: \mu = 95\%$ when H_0 is true
Consequences: The client may receive monetary compensation he is not entitled to. Disciplinary action may be taken on the broker unnecessarily.
Probability: same as $\alpha = .05$ 5% chance

Type II Error: accept H_0 when H_0 is false
Consequences: The judge may rule against the client when he is due compensation.
Probability $= .17$ Power $= 1 - .17 = .83$