

10.2 Estimating a Population Mean

Remember: Conditions for Inference about μ

SRS

Normality

Independence $N \geq 10n$

If the population standard deviation, σ , is unknown you must use the sample standard deviation, s .

Standard deviation of sampling distribution

$$= \boxed{\frac{s}{\sqrt{n}}} = \underline{\text{Standard error}} \quad \text{S.E.}$$

Sampling Distribution

	<u>σ known</u>	<u>σ unknown</u>
Standard error:	$\frac{\sigma}{\sqrt{n}}$	$\frac{s}{\sqrt{n}}$
distribution:	normal z-distribution	degrees of freedom t-distribution df = n-1
Confidence Interval:	$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$	$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$

(slide 8) Density Curves

Similar to N: symmetric to 0, bell shaped, single peak

Contrast to N: larger spread, more area in tails,

As df increases, curve approaches N

* When df does not show up in Table C, use the greatest value less than df *

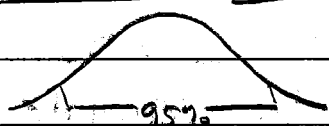
Finding t^* Table C

95% CL with $n=10 \Rightarrow df=9 \quad t^*=2.262$

80% CL with $n=7 \Rightarrow df=6 \quad t^*=1.440$

99% CL with $n=51 \Rightarrow df=50 \quad t^*=2.678$

Calculator: 2nd Vars invT (area, df)



$$n=10$$

$$z=1.96$$

t^*

area to the left = .975

$$df=9$$

$$t=2.262$$

$$\text{area} = \frac{1+c}{2}$$

2

Steps for Confidence Interval with σ unknown

① one sample t^* Confidence interval for mean context

② SRS - SRS, independent, $\sim N$

③ Math $\bar{x} \pm t^* \left(\frac{s}{\sqrt{n}} \right)$; $df: n-1$

④ Interpretation in Context

Example 1 (Slide 19 - Day 1)

① one-sample t confidence interval for mean mileage (one way) for all college students to school

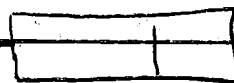
② (Simple random sample is stated): $N \geq 10n=10(30)=300$. Since there are ≥ 300 college students that commute we can assume independence.

10.2 cont'd

Example 1 (cont'd)

Data: L,
Graph
Box-whisker

② A graph of the distances is $\sim N$, so we can assume the population of distances is $\sim N$.



1VAR Stat

③ $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ df $n-1$ 95% CI df=29

$t^* = 2.045$

STAT-TESTS

$18.867 \pm 2.045 \left(\frac{9.989}{\sqrt{30}} \right)$

$\bar{x} = 18.867$

TInterval: Data

(15.137, 22.597)

$s_x = 9.989$

df = 29

④ Based on this sample, we are 95% confident that the true mean distance driven one-way to college by all students is between 15.137 and 22.597 miles.

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Review Ch. 5

Matched pairs block design - two subjects are paired and either:

① each subject receives both treatments

② one subject of the pair receives the treatment

Paired t procedures - to compare the responses to the two treatments in a matched pairs design or before-and-after measurements on the same subjects, apply

one sample t procedures to the observed differences.

The parameter μ in a paired t procedure, μ_D

- the mean difference in the responses to the two treatments within matched pairs of subjects in the entire population (subjects are matched in pairs)

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- the mean difference in response to the two treatments for individuals in the population (when the same subject receives both treatments)

- the mean difference between before-and-after measurements for all individuals in the population (for before-and-after on the same individuals)

Example 2 (slide B-9) Is Caffeine Dependence Real

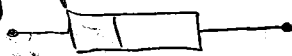
① Parameter: One-sample t confidence interval for the mean difference in depression scores for this population. $\mu_D = \mu_{\text{placebo}} - \mu_{\text{caffeine}}$

② Conditions (Simple random sample of differences in depression scores is stated). $N \geq 10n = 10(11) = 110$.

SIN

(B-652)

It is safe to assume there are more than 110 people addicted to caffeine, so the condition of independence is satisfied. A graph of the differences is approximately normal.



We will use one-sample t -procedures to construct the CI for μ_{diff} since σ is unknown.

10.2 cont'd

(3) Calculations: $\bar{x} = t^* \left(\frac{s}{\sqrt{n}} \right)$ $t = 1.812$
 $7.364 \pm 1.812 \left(\frac{6.918}{\sqrt{11}} \right)$ $\bar{x} = 7.364$
 $df = 11 - 1 = 10$
 $(3.583, 11.144)$

(4) Interpretation: Based on this sample we are 90% confident that the true mean difference in depression scores (placebo - caffeine) for the population is between 3.583 and 11.144 units.

Random selection of individuals allows us to generalize the results of the study to a larger population

Random assignment of treatments to subjects in an experiment lets us investigate whether there is evidence of a treatment effect, which might suggest the treatment caused the difference

Procedures not strongly affected by lack of normality are called robust.

The t procedures are not robust against outliers, because \bar{x} and s are not resistant to outliers

The t procedures are robust against non-normality when there are no outliers; CLT

