10.1 Confidence Intervals

Statistical Inference - provides methods for drawing conclusions about a population from the data

Example 10.1

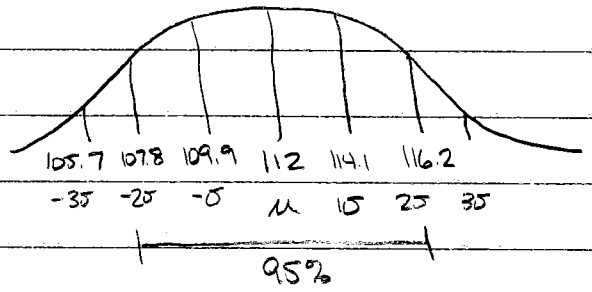
IQ scores of Big City University SDOO freshman

($\hat{p}$) parameters: $\mu_S = 112$ $\sigma_S = 15$

($\bar{x}$) Sample of 50 $\mu_{\bar{x}} = 112$ $\sigma_{\bar{x}} = \frac{15}{\sqrt{50}} = 2.1$

Test before using $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
$N \geq 10(n)$
Shape $\sim N$ b/c $n \geq 30$

normal distribution



* In 95% of all sample means will be within 4.2 of the population mean μ. (107.8, 116.2)
* The sampling distribution of $\bar{x}$ signifies how big the error is likely to be when $\bar{x}$ is used to estimate μ.
* The interval $\bar{x} \pm 4.2$ captures μ in 95% of samples
* "We are 95% confident that the unknown mean IQ score for all Big City freshman is between 107.8 and 116.2."

Resulting in two possibilities:

1. The interval between 107.8 and 116.2 contains the true μ.
2. Our SRS was one of the few samples for which $\bar{x}$ is not within 4.2 points of the true μ. Only 5% of all samples give such inaccurate results.

confidence interval: $\bar{x} \pm 4.7$
confidence level 95% (success rate)
margin of error ± 4.2

* The confidence level does NOT tell us the probability that a particular confidence interval captures the population parameter.

Exercise 10.2
(a) Incorrect; The probability is either 0(false) or 1(true)
(b) Incorrect; $\bar{x} \pm$ margin of error means $\bar{x}$ will always be in the center
(c) Incorrect; The distribution is used to provide an interval that captures the true mean
(d) Incorrect; only meant to capture the mean
(e) correct

Exercise 10.3
No The student is misinterpreting 95% confidence. This is a statement about the mean score of all young men, not individual scores.

Exercise 10.5
(a) $51\% \pm 3\%$ (48%, 51%) (b) 51% is only from one sample
(c) 95% confidence means the interval was found using a procedure that produces correct results. 95% of the time
(d) non-response, voluntary response

Interpretation Script: "Based on this sample, I am ___% confident that the true _____ of all _____ is between ___ and ___."

Sample Size for Desired Margin of Error

Solve for n in the inequality: $z \cdot \frac{\sigma}{\sqrt{n}} \leq M$

(M) margin of error: $z \cdot \frac{\sigma}{\sqrt{n}}$ or $z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Authors pg. 636

10.1 cont'd

Conditions for Constructing Confidence Intervals

must be checked

SIN

SRS - data comes from an SRS of the population

Independence - individual observations are independent. When sampling without replacement, $n \geq 10n$

Normal - Sampling distribution of $\bar{x}$ must be $\sim N$

means: $n \geq 30$ proportions: $np \geq 10$ and $n(1-p) \geq 10$

Table A - Confidence levels

Confidence level	Tail area	z (critical values)
90%	.05	1.645
95%	.025	1.960
99%	.005	2.576

$z = 1.96$ is more accurate than $z = 2$ for 95% (68-95-99.7 rule)

Confidence Interval for a Population Mean (σ known)

Example 10.5 Constructing a confidence interval

C confidence interval for $\mu = \bar{x} \pm z \cdot \frac{\sigma}{\sqrt{n}}$

Inference Toolbox

1. Parameters - Identify population
2. Conditions - Choose correct procedure (SIN)
3. Calculations - CI = estimate ± CV (standard error)
4. Interpretation - conclusion, connection, context